Kinetics and Design of Semi-Compliant Grid Mechanisms

Jonas Schikore[1,*], Eike Schling[2,*], Thomas Oberbichler[3], Anna M. Bauer[3]

[1] Technical University of Munich, Faculty of Architecture, Chair of Structural Design,

[*] Corresponding authors e-mail: Jonas.schikore@tum.de, schling@hku.hk

[2] The University of Hong Kong, Department of Architecture, Pokfulam Road, Hong Kong.

[3] Technical University of Munich, Department of Civil, Geo and Environmental Engineering, Chair of Structural Analysis, Arcisstr. 21, 80333 Munich, Germany

Abstract

The research of transformable structures has fascinated architects, engineers, and mathematicians (e.g. Piniero, Hoberman, Otto, Finsterwalder). Their potential to adapt to environmental conditions and user's needs or to aid the erection process has produced beautiful and complex designs (e.g. Hoberman sphere, Multihalle Mannheim). We can distinguish between conventional, discrete rigid-body-mechanisms and so-called compliant mechanisms, which utilise the elasticity of members to perform controlled elastic deformations (Howell 2001). Recent developments in elastic gridshell construction have used this kinetic behaviour for the construction process and transformation (X-Shells, G-Shells, Asymptotic Gridshells).

This paper aims to unravel the categories of such kinetic gridshells, and presents fundamental principles of semi-compliant quadrilateral grid structures with uniaxial rotational (scissor) joints built from initially straight, continuous beams. We use specific lamella profiles that restrict the elastic deformability, disabling at least one of the three local bending axes. Depending on the orientation of the profiles, we can categorise three families – doubly ruled (straight), geodesic and asymptotic networks – each exhibiting distinct kinetic properties with limited degrees of freedom. We verify our theory using a matrix of 3 x 2 basic/fundamental grid configurations. By controlling the structure's parameters, we can design their shape and behaviour. Introducing the curvature-square diagram further allows us to understand and predict their kinetic performance. We present architectural applications of both experimental and built structures and apply our theory to a novel design for the kinetic umbrella, a transformable asymptotic gridshell.

Keywords: transformable structures, gridshells, active bending

1 Introduction

Transformable structures show a wide range for architectural designs, from adaptable façade solutions to retractable roofs or bridges. In contrast to conventional rigid body mechanisms, compliant mechanisms can perform smooth transformations and supersede hinges (Howell 2001).

In this paper we focus on a hybrid (or semi-compliant) typology of **elastic grid mechanisms**, combining scissor joints with elastic deformation in a one-layered, doubly curved, quadrilateral grid. Their geometric and mechanical parameters can be controlled, to create a predetermined kinetic behaviour.

1.1 Related work

Even though we approach this topic from an engineer's point of view, we can find fundamental similarities with the theory of **differential geometry**, some of which have been described early on by the mathematician Finsterwalder (1899). He sets up the analogy of curvature lines on surfaces and physical members within grid structures and describes a series of networks with specific deformation behaviour.

The most prominent architectural examples are the timber gridshells first developed by Frei Otto. During the construction of the Multihalle in Mannheim (**fig. 1**), Otto took advantage of the elastic deformability of equilateral grids (Burkhardt and Bächer 1978; Happold and Liddell 1975). However, these gridshells do not feature a directed movement, because the slender, square timber profiles do not sufficiently restrict their kinetic behaviour.

Figure 1: Multihalle in Mannheim: Application of elastic grid deformation (erection). The final shape is determined by scaffolding, without directed deformation behaviour.

Increasing the bending stiffness of the profile's $x-$, $y-$ or $z-$axis, may restrict its bending behaviour and create a more controlled movement. Schling (2018) used this concept in the design and construction of a steel gridshell using asymptotic curves (**fig. 2c**, see also **sec. 4**). Currently, investigations on deployable grid structures called "X-Shells" (**fig. 2b**) and the subclass limited to geodesic grids called "G-Shells", (**fig. 2a**) deal with compliant transformations (Soriano et al. 2019; Pillwein et al. 2020; Panetta et al. 2019) .

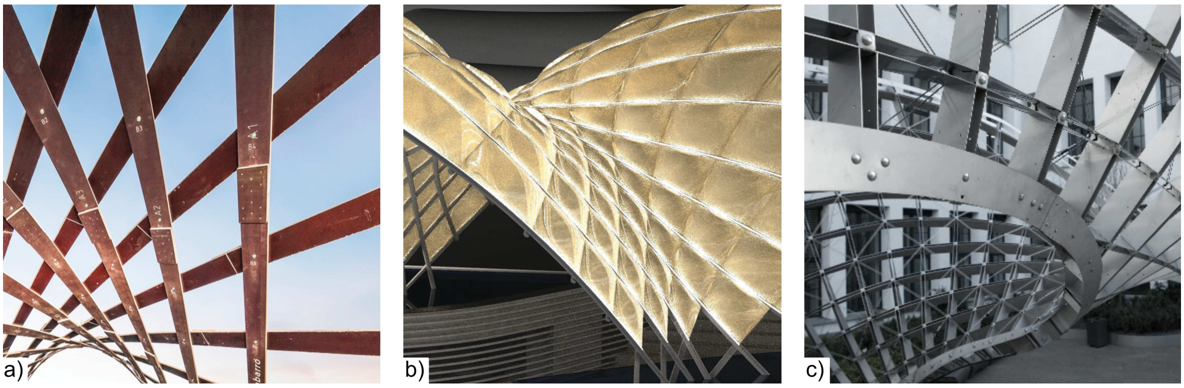

Figure 2: (a) “G-Shell”, Nuela Pavilion by E. Sorriano et. al., (b) Visualisation of a “X-Shell” by M. Pauly et al., performing compliant transformations, (c) Asymptotic gridshell - Inside/Out by E. Schling et al.

1.2 Contribution

This paper aims to unravel such semi-compliant grids and bring clarity to their kinetic behaviour by identifying the dependencies of their individual geometric and mechanical parameters. We give an overview of fundamental principles of semi-compliant grid mechanisms and classify them by the geometric curvature network they represent – doubly ruled, geodesic, or asymptotic. We present practical numerical methods to simulate and analyse such mechanisms making them accessible for designers. We introduce the curvature-square diagram as a specific characteristic and design tool for predetermined mechanisms.

We present applications in architectural design, looking at built gridshells and their erection process, a future application for a transformable façade system, and a series of physical models – results of an experimental design studio.

Finally, we apply our theory to design the Kinetic Umbrella, a transformable pavilion for the AAG2020.

2 Theory

We are interested in quadrilateral grid structures that combine elastic (initially straight) beams with hinged joints, and thus create a class of semi-compliant mechanisms, with the potential to perform complex spatial transformations. We can control and prescribe the geometry and behaviour of semi-compliant grid mechanisms, based on several geometrical and mechanical parameters. We specifically look at hinge conditions, the beam sections and orientation, the resulting network, as well as the supports, actuation, locking and natural limits.

Hinges. are part of semi-compliant mechanisms. The spatial deformation of the grid is dependent on the change of the rhombus cells' angles. Therefore, rotational freedom around the joint's $z-$vector is necessary. If the rotational degree of freedom is limited to this axis at every joint, the beams will keep their orientation normal to the base surface throughout the transformation.

Beam sections. The flexibility of a beam can be restricted either to allow twisting predominantly around the local $x-$axis (e.g. L-Sections, T-sections or cross-shaped sections) or to allow additional uniaxial bending around the local $z-$, or $y-$axis. We can thus define three representative sections, which we call a) cross-shaped section, b) tangent lamella and c) normal lamella, as shown in fig. 3.

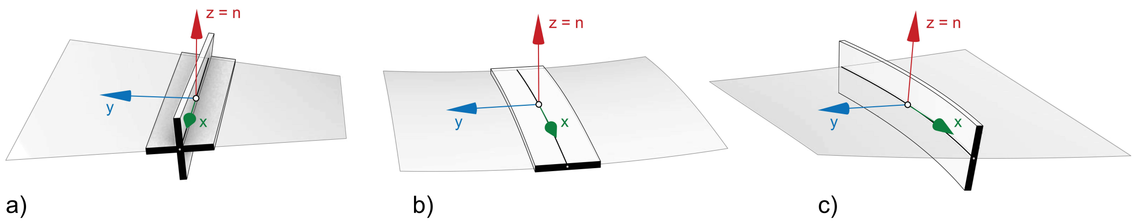

Figure 3: Basic representative sections on curvature lines: (a) cross-shaped section, (b) tangent lamella, (c) normal lamella.

Other restricted sections, such as a beam, that allows biaxial bending, but no torsion are mechanically nearly impossible, and will thus not be considered in the subsequent investigation.

Network. If we consider the profiles to be continuously aligned with the design surface, so that the local **z**-direction of the profiles is equivalent to the surface normal **n**, the beams will inevitably follow certain geometrical paths, reflecting their own flexibility, and will thus be restricted to certain geometric networks (Schling et al. 2018).

Cross-shaped sections will remain straight and thus form doubly ruled networks. Their design space is restricted to the hyperbolic paraboloid (HP) or Hyperboloid (RH).

Tangent lamellas will not deform sideways and thus follow geodesic paths. They are the least restrictive and can be generated on any smooth surface, both synclastic and anticlastic. Geodesic curves can be drawn from any point in any direction and thus allow for maximal input from the designer.

Normal lamellas will follow the path of zero normal curvature, called asymptotic curves, on anticlastic surfaces. Such networks are dependent on the surface curvature and naturally form quadrilateral networks.

Supports can further restrict the degrees of freedom of the system. They range from basic types, such as fixed or rigid constraints, linear sliding or uniaxial hinge to more complex support conditions such as curved sliding supports or biaxial hinge. They may compensate undesirable flexibilities of the grid structure.

Actuation principles might be rotational or translational, force or path controlled, internally or externally applied, whichever is suitable. Furthermore, it is good practice to design specific **locking** mechanisms within the structure, via supports or relative constraints. They may stiffen the structure at a chosen position and/or define the transformation range.

Besides these design parameters, compliant mechanisms have natural **limits**, such as self-collision or their yield strength.

3 Methods

To design and evaluate compliant mechanisms, we need to simulate their kinetic behaviour. Doubly ruled grids can be described by rigid body transformation and its geometric relations. For structures and mechanisms that are dependent on bending, numerical methods are necessary.

In the following section, we first retrace a clean geometric approach for simple doubly ruled grids and later describe numerical simulation methods and workflows, using the Finite Element Method (FEM) and Isogeometric Analysis (IGA).

3.1 Rigid body transformation

Rigid body transformation can be described using geometric rules and relations. If we consider grids of double rulings, namely the hyperbolic paraboloid and the hyperboloid, some kinematic relationships using full rotational joints are explained by Hilbert and Cohn-Vossen (2011) and Maden and Korkmatz (2017). Let us briefly recapitulate the kinematic relations:

The geometry of the hyperbolic paraboloid (fig. 4a) is defined by the edge length L and the projection of this edge length d into the XY-plane. This allows the computation of the four corner points:

$$A_{1,2} = \left[0 \; \pm d \sin\left(\frac{\alpha}{2}\right) \; \frac{1}{2}\sqrt{L^2 - d^2} \right]; \; B_{1,2} = \left[\pm d \cos\left(\frac{\alpha}{2}\right) \; 0 \; -\frac{1}{2}\sqrt{L^2 - d^2} \right] \quad (1)$$

Its grid results from a uniform subdivision. Changing the angle α does not influence the edge lengths of the segments (measured between two nodes). It can therefore be used to describe the mechanism. The angle must be in the range from 0 to 180°. It is therefore expressed by the normalised parameter t.

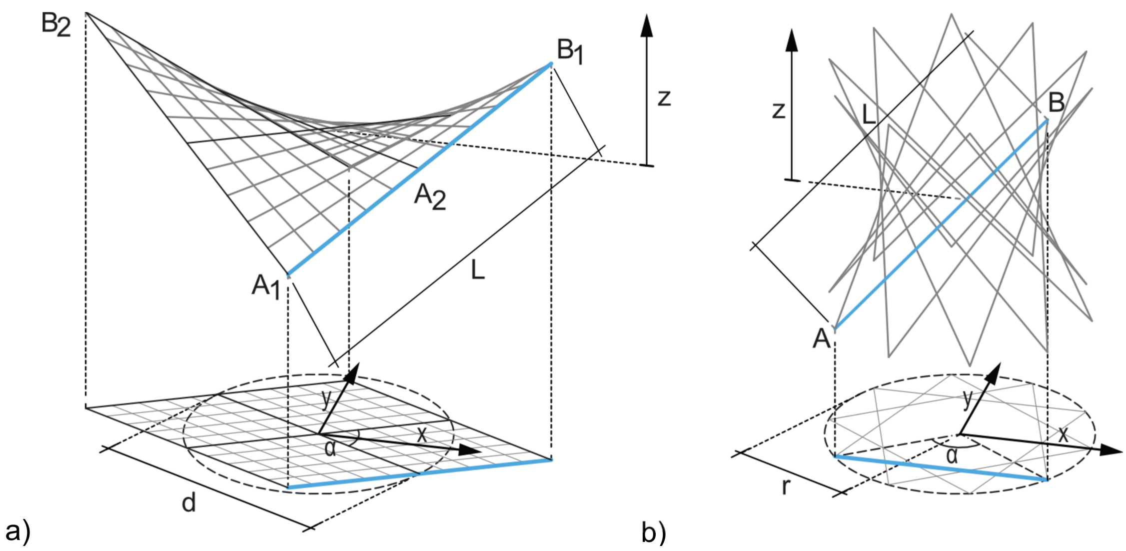

Figure 4: Rigid body motion of a grid structure: (a) hyperbolic paraboloid, (b) hyperboloid.

$$\alpha = t \cdot 180° \tag{2}$$

The hyperboloid (**fig. 4b**) is defined by the edge length L and the angle α between the projected end points of an edge. For α the following must apply:

$$\alpha = q \cdot \pi \quad q \in \mathbb{Q} \tag{3}$$

With the following formula, the spatial position of the end points can be calculated:

$$A, B = \left[\pm r \sin\alpha \quad r\cos\alpha \quad \pm\frac{1}{2}\sqrt{L^2 - 4r^2\sin^2\alpha}\right] \tag{4}$$

Its grid results from rotation and mirroring. Changing the radius r allows controlled movement of the geometry without changing the lengths of the segments. We describe the movement again using the normalised parameter t:

$$r_{max} = t \cdot \frac{L}{2\sin\alpha} \tag{5}$$

This calculation gives us the system lines of the doubly ruled grids. The orientation of the profiles is assumed to be aligned to the normal direction of the doubly ruled grid (or of the base surface). We can now measure the torsion to get a complete picture of the deformed structure at any state.

3.2 Numerical simulation

FEM is typically used for static calculations. Tools are widely available and well developed. Within FEM, a curved beam is approximated by a polygon. To calculate a grid structure, starting from given curves, we use a systematic workflow shown in fig. 5. We polygonise the curved beam and measure the angles at the kinks. We apply discrete rotational restraints at each kink, resetting the beam to an initially continuous member.

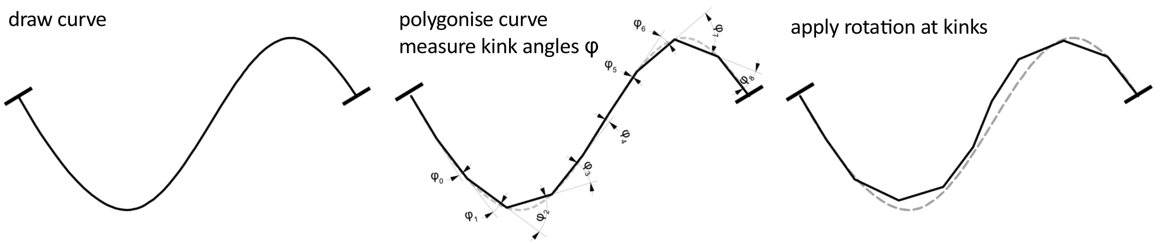

Figure 5: Workflow modelling curve from a deformed start geometry using conventional FEM.

For spatial cases, all three local angles of the kinks of the polygons are considered respectively. Other elements, supports, loads, etc. of the calculation model are treated conventionally. This workflow allows the use of well-developed FEM-software which gives access to software features, especially valuable for practical applications.

IGA, in contrast to classical FEM approaches, omits the approximation of the curve by a polygon. In this approach, we start the analysis from an already deformed structure. The moments for the potential energy are computed by the curvature and torsion of the curve (Bauer et al. 2019). This geometrically nonlinear problem is then solved like the conventional FEM approach.

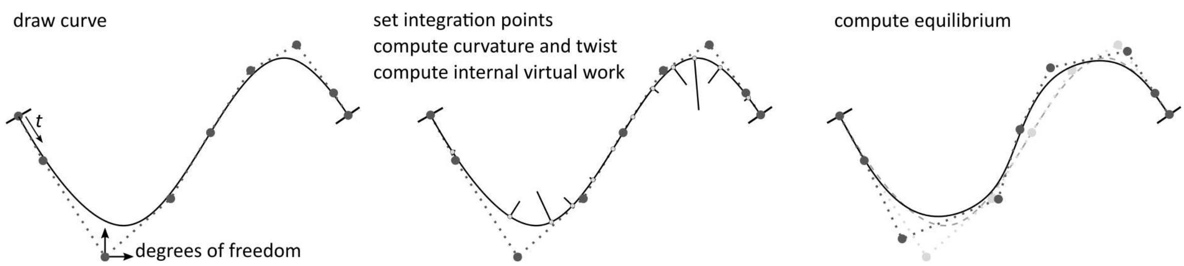

Figure 6: Workflow modelling curve from a deformed start geometry using IGA.

The IGA approach allows a smooth workflow for grid structures since the NURBS curve from CAD can be defined independently from connection points. The scissor hinges are implemented as additional coupling elements with a defined rotational axis.

3.3 Evaluation methods

If the structural system is set up correctly, it follows a predetermined transformation, and its compliant elements perform predetermined deformations. The mechanism is then characterised by an individual internal energy profile. To evaluate the **internal energy**, let us recall some mechanical basics. The energy stored in an elastically deformed beam can be calculated as the sum of portions due to elongation, shear and bending around the local axis x,y and z (torsion and bending). For structures with large bending and torsional deformations, the energy portions due to elongation and shear are comparatively miniscule. In this case, the energy function can be reduced:

$$\Pi_i = \frac{1}{2}\left[GI_t \int_C \kappa_x^2 ds + EI_y \int_C \kappa_y^2 ds + EI_z \int_C \kappa_z^2 ds\right] \tag{6}$$

The stiffness of the beam is expressed by the constant material parameter E (Young's modulus) and G (shear modulus) and the profiles moment of inertia I. Any structural system under restraint will find its shape at a global or local minimum of internal energy Π_i. Note, this function does not include internal energy portions due to "helix torsion" (Lumpe and Gensichen 2014).

Curvature-square graph. If the mechanisms transformation path is predetermined, the material and profile's stiffness (E , G and $I_{t,y,z}$) scales the energy portions while the curvature-square ($\kappa^2_{t,y,z}$) summed up over the beam length is specific to the mechanisms state. We introduce as a parameter defining the structure's state and draw the curvature-square graphs which displays the kinetic behaviour of the system. This graph gives valuable information to understand, design, and control such mechanisms. For a lamella section, only two components are relevant: **torsion** (κ_t^2) and **uniaxial bending** ($\kappa^2_{y/z}$). Some exemplary graphs are shown in fig. 7.

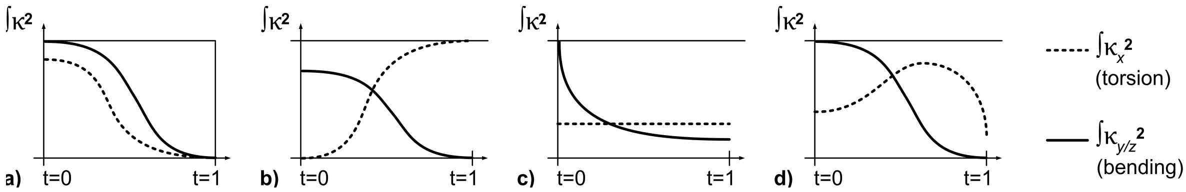

Figure 7: Exemplary curvature-square graphs, describing the basic kinetic behaviour of a mechanism. These graphs display the geometric factor of the structure's internal energy during transformation.

From these graphs we can derive important characteristics of the kinetic behaviour. Let us derive some characteristics from the exemplary graphs in fig. 7:

a The natural state of the mechanism is at $t = 1$, independent of the profile or material stiffnesses. Additionally, there is a state of no internal energy at $t = 1$.

b Bending and torsion work in opposite directions. The natural state can be controlled by the stiffness relations.

c Torsional stiffness does not influence the natural state, which is at $t = 1$ for any material or section. At $t = 1$, the curves are almost flat, which indicates, that nearly no forces are necessary to perform transformation

d There is a natural state at $t = 1$, but there might be another local minimum anywhere before, which could create a snap-through mechanism. The position of the second natural state can be controlled by the stiffness parameters.

Besides internal strain energy, the potential energy through **self-weight** also has a decisive impact on the structures natural state, especially on larger scale architectural applications. The energy function can thus be extended by the portion: $(\rho A \int_C h ds)$, where the height is measured from a reference level, e.g., the level of supports. We will implement the portion of self-weight only in our practical applications, the Kinetic Umbrella (**sec. 5.3**).

4 Analysis

Let us look at some general characteristics regarding the transformation of doubly ruled, geodesic, and asymptotic curvature networks and systematically explore their kinetic behaviour. For this purpose, we perform compliant transformations for basic archetypes of open and closed (rotational) grids to evaluate their kinetic characteristics, using the curvature-square graph. For this qualitative analysis, we will normalise the curvature-square graphs. **Figure 8** shows our matrix of analysed structures. All open structures are doubly symmetric, all rotational structures consist of identical elements, all are based on a 20 x 20 beam layout.

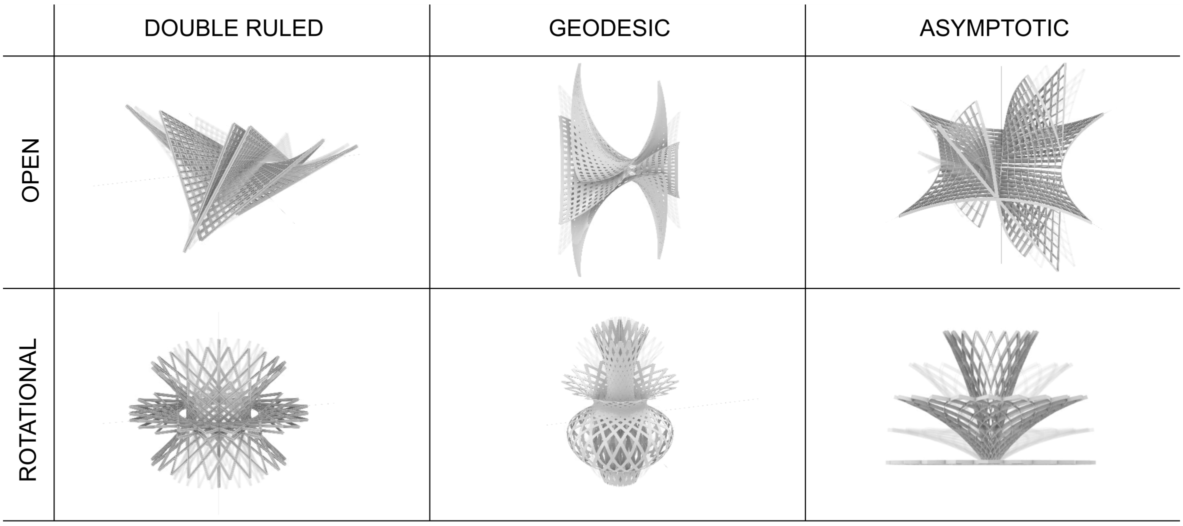

Figure 8: Overview of basic semi-compliant mechanisms. These categories each possess a characteristic kinetic behaviour. We distinguish between doubly ruled, geodesic and asymptotic grids on open or rotational base surfaces.

This analysis targets on restricted profile deformations using numerical methods. Therefore, high stiffness parameters are applied for axial and restricted local bending deformations. Materials are considered to be infinitely linear elastic. The transformations are actuated in a minimal way, as to not restrict the shape or kinetic behaviour.

4.1 Doubly ruled grids

We set up a hyperbolic paraboloid (HP) and a rotational hyperboloid (RH) for a geometric analysis of the curvature-squared values (**fig. 9**) using a 20×20 beam network. We can assume the beams to be sufficiently orientated normal to the network's base surface at any position and state. This is valid for an infinitely dense grid.

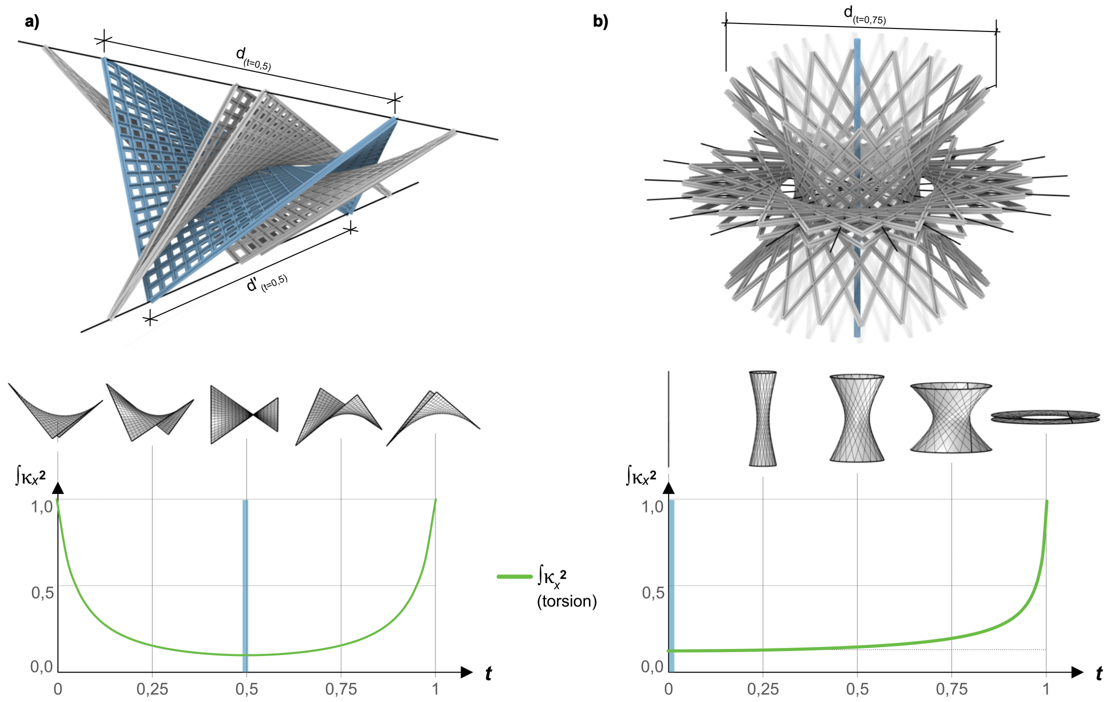

Figure 9: Parametric simulation and curvature-square graphs of mechanisms on doubly ruled grids: (a) hyperbolic paraboloid (open surface), (b) hyperboloid (rotational surface).

We observe some geometric aspects from rigid body transformation: Both can be "folded" planar with overlapping lines; the height of the HP stays constant and the paraboloid can be packed into a line.

Besides the rigid body transformation, the beams are twisted, adding a compliant behaviour. We visualise the torsional energy using the curvature-square diagram (see **fig. 9**) along the parameter t. The graphs display the natural state (marked blue) of a doubly symmetric HP structure when the corner angles are equal and of the RH, when it is in a linear position, along the rotational axis. The extremities are defined not only by a natural collision of the grid members, it also creates an infinite increase in energy due to torsion, along the folding line.

Note: to visualise the symmetric character of the HP, the parameter t is referenced to the dimension d for $t = [0; 0.5]$ and d' for $t = [0.5; 1]$ respectively. For the RH and all other analysed mechanisms, the parameter t is referenced to the dimension d.

4.2 Geodesic grids

Geodesic grids have a higher degree of flexibility compared to doubly ruled networks. We identify two possible transformations: A semi-compliant deformation including change of the angles, and a purely compliant deformation behaviour, namely inextensional deformation (Williams 2014), without changes in angles. Differential geometry provides useful approaches to set up geodesic networks for suitable semi-compliant mechanisms (Williams and Soriano 2015). Unlike for doubly ruled grids, in the following cases (**sec. 4.2** and **4.3**), we set up a structural model and use the IGA method to evaluate the mechanism.

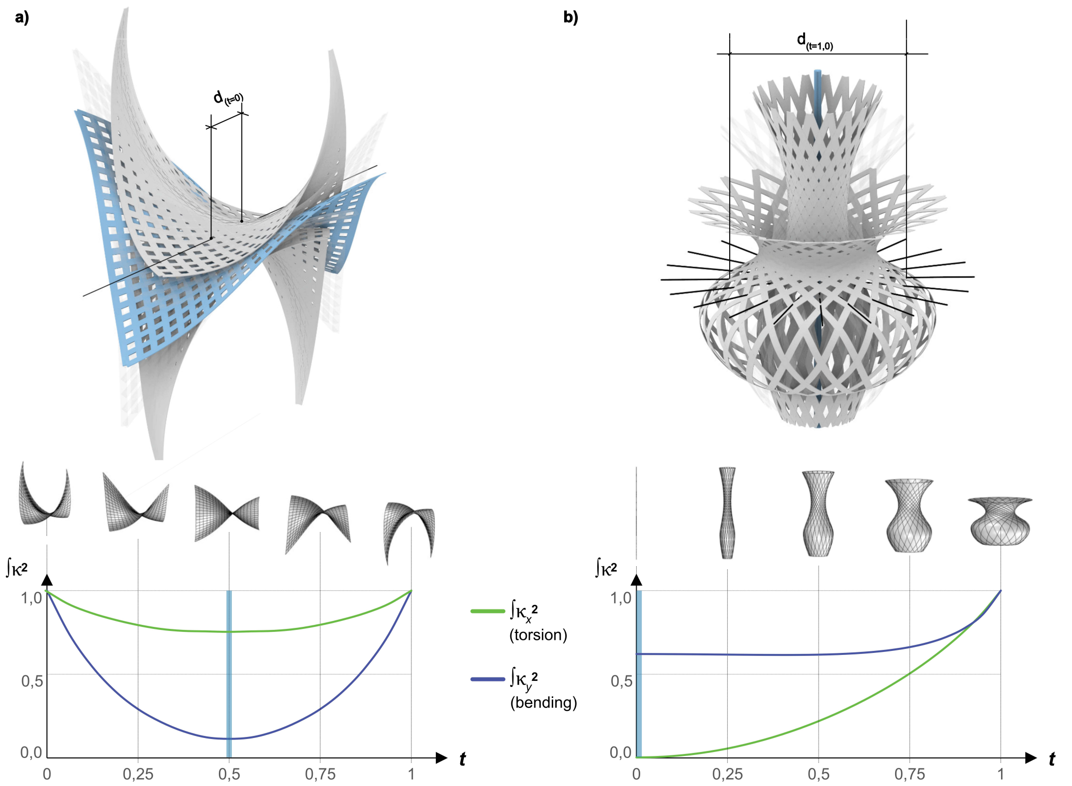

Figure 10: Numerical simulation and curvature-square graphs of geodesic mechanisms: (a) open surface, (b) rotational surface.

Setup. For an open network, we create a doubly symmetric freeform surface, find the edge lines, subdivide these, and interconnect the points of subdivision (**fig. 10a**). For the rotational grid, we create any rotational surface, find a geodesic curve between points on the top and bottom edges and copy the curve radially. The second family of curves is generated by mirroring (**fig. 10b**). The grid structures are initially supported statically determinated. We define actuation, in this case, by forcing selected points to keep a defined distance d . The nodes for actuation are marked in **fig. 10**, where the dimensions are located. The mechanisms are actuated, by moving these nodes, changing the dimension d respectively. This actuation is designed to not restrict the overall shape and behaviour of the grids, but merely activate their natural transformation.

Results. The observed transformation range is in between natural limits. The structures perform spatial transformations until the first rhombi are nearly closed. One natural limit of the rotational mechanism is a "packed" state (line bundle), another spatial.

The curvature-square diagram shows that both systems have a clear minimum of internal energy. We also observe the analogies to the hyperbolic paraboloid and hyperboloid. The graph shows that there is no possibility to control the natural state by stiffness parameters.

4.3 Asymptotic grids

Unlike geodesic curves, asymptotic curves are driven by the curvature only and result in quadrilateral networks on anticlastic surfaces. Structurally, an upright lamella orientation leads to a girder like load-bearing behaviour, a useful quality, because it counteracts inextensional deformation. On the other hand, asymptotic grids have a low stiffness in-plane that can, however, be controlled by using appropriate support settings. In most cases, the deformation behaviour allows a planar state (unless physical limits, e.g., overlapping, are reached). This is a valuable quality for construction and assembly. For a closer analysis of the kinetic behaviour, we set up two exemplary structures and run numerical calculations using IGA.

Setup. For a grid on an open surface, we use the same hyperbolic paraboloid as used for the linear grids (see **fig. 11a**). Note, that the generating straight lines of the doubly ruled networks are also both geodesic and asymptotic lines. The open is statically determinate with a rigid support at the centre of the grid. The upper and lower corner pairs are pushed or pulled horizontally. Our simulation does not reach natural limits, but a collision obviously occurs for $d = 0$. The simulation is stopped when the flat state is reached.

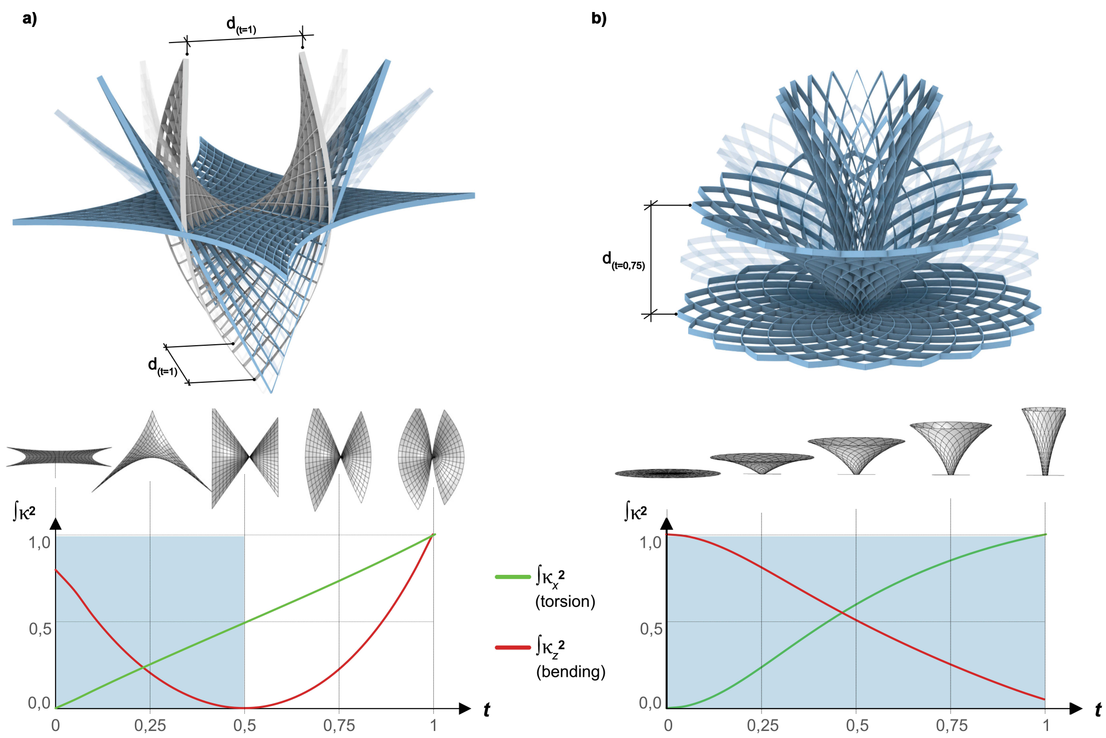

Figure 11: Curvature-square graphs of asymptotic mechanisms: (a) open surface, (b) surface of revolution.

To create an asymptotic rotational model, we find an asymptotic curve on a chosen anticlastic rotational base surface, and generate the network (**fig. 11b**). We apply upright lamellas and the grid is supported at the bottom nodes. The mechanism is actuated by pushing or pulling the outermost nodes towards the base plane (direction to the axis of revolution). The transformation spans the natural limits between the closed funnel shape and the flat state.

Results. We observe a similarity between the two mechanisms. The curvature-square graphs display a contrary relation of bending and torsion, going from flat to spatial. In a flat state, there is obviously no torsion, but possibly bending. Going into a spatial shape reduces the bending energy for both cases first while torsion is induced. In this area (shaded blue), the minimum energy state can be controlled by the beam's stiffness parameters. The hyperbolic paraboloid marks a turning point in the structure's kinetic behaviour of the open structure, limiting the tenable deformation range. The rotational mechanism's energy terms act contrary over the whole observed range. Note that any flat state marks a symmetry plane and thereby a possibly global or local energy minimum or maximum of the mechanisms. Both could also be performed in the opposite direction.

5 Architectural Application

In the following chapter we will introduce architectural applications for semi-compliant grid mechanisms, focusing on the category of asymptotic networks. The projects range from built gridshells (simplifying the erection process), to designs and physical models of transformable structures. We use the theory presented in this paper to design a large scale, kinetic umbrella, a transformable structure for the AAG2020.

5.1 Erection process for Gridshells

The first architectural application of a compliant asymptotic grid mechanism was discovered and implemented for the construction of the **Inside/Out** -Pavilion in 2017 (**fig. 12**). The structure is built from straight steel lamellas that are slotted together to form scissor joints, thus creating kinetic behaviour with similar characteristics to the open asymptotic structure (**fig. 11a**). This behaviour was used to prefabricate nine curved segments. The segments were assembled flat, hoisted onto a cross-shaped support and deformed simply by hand (Schling 2018). The designed shape emerges naturally and is locked at final position. This project served as the inspiration for further investigations.

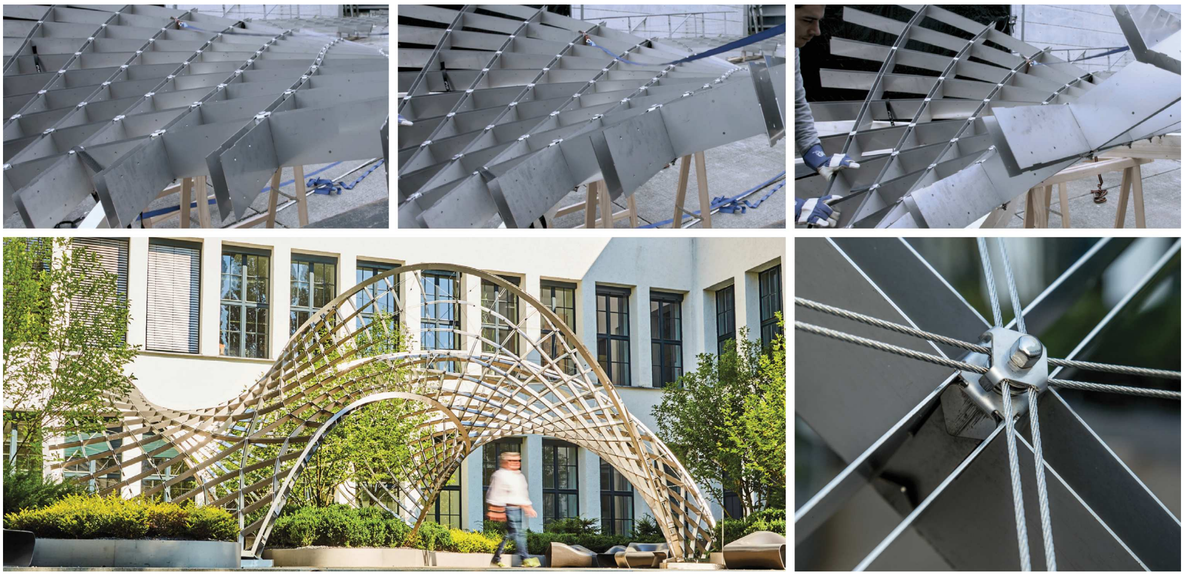

Figure 12: The INSIDE/OUT Pavilion in Munich (bottom, left). The lamella grid and scissor joints restrict the kinetic behaviour to be used as a compliant mechanism during construction (Top row). Nine segments were deformed by hand following the characteristics of an asymptotic disc (see **fig. 11a**). The final geometry is locked by tightening the washers at each joint, and adding diagonal ties (bottom, right).

The **Canopy of the Hotel Intergroup in Ingolstadt** was completed in December 2019 (see **fig. 13**). The roof is created from four symmetric leaf-like segments, which were separately prefabricated, and deliberately designed for an inverted "pull-up" strategy (Schikore et al. 2019). The stainless-steel lamellas were interlaced via slots and temporarily secured with steel brackets. Each segment was then pulled-up by a crane into its design shape. The process is similar to creating an inverted hanging model but following a compliant mechanism. The deformed grid was fitted within a rigid steel frame and then welded at each joint. The four segments were finally assembled on site to become one structural unit. The structure was fitted with two arch supported membranes on top.

Figure 13: The Canopy of the Intergroup Hotel in Ingolstadt. Four symmetric lamella grids were assembled and deformed using an inverted pull-up strategy. The elements were then welded and assembled on site.

The erection process performed in these projects provided valuable insights for the development of transformable applications. The potential energy of the structure due to self-weight is high compared to the strain energy that could be stored within the material limits. A self-induced lifting of these steel structures is nearly impossible.

5.2 Research designs

Initial tests of semi-compliant mechanisms have been conducted through digital and physical research prototypes, as part of experimental design studios. In the following, we present three design-research projects aiming to bring this technology into building practices.

An experimental design studio conducted in 2019 investigates compliant mechanisms through physical prototyping. The resulting timber models shown in **fig. 14** display the great potential for architectural design, ranging from modular roof systems to domes. **Figure 14a** shows a sequence of transformation of an open asymptotic structure. Six modules create the primary structure of a transformable roof system. **Figure 14b** shows a transformable dome. The mechanism is derived from C. Hoberman's well known "Iris Dome". This semi-compliant variation allows a similar transformation, but using only uniaxial joints and keeping the base circumference constant.

Figure 14: Experimental designs of semi-compliant grid mechanisms: (a) "Active Grillage" – Asymptotic mechanism on a "disc"-structure, (b) "EXX-Dome" – Geodesic variation of C. Hoberman's "Iris Dome" .

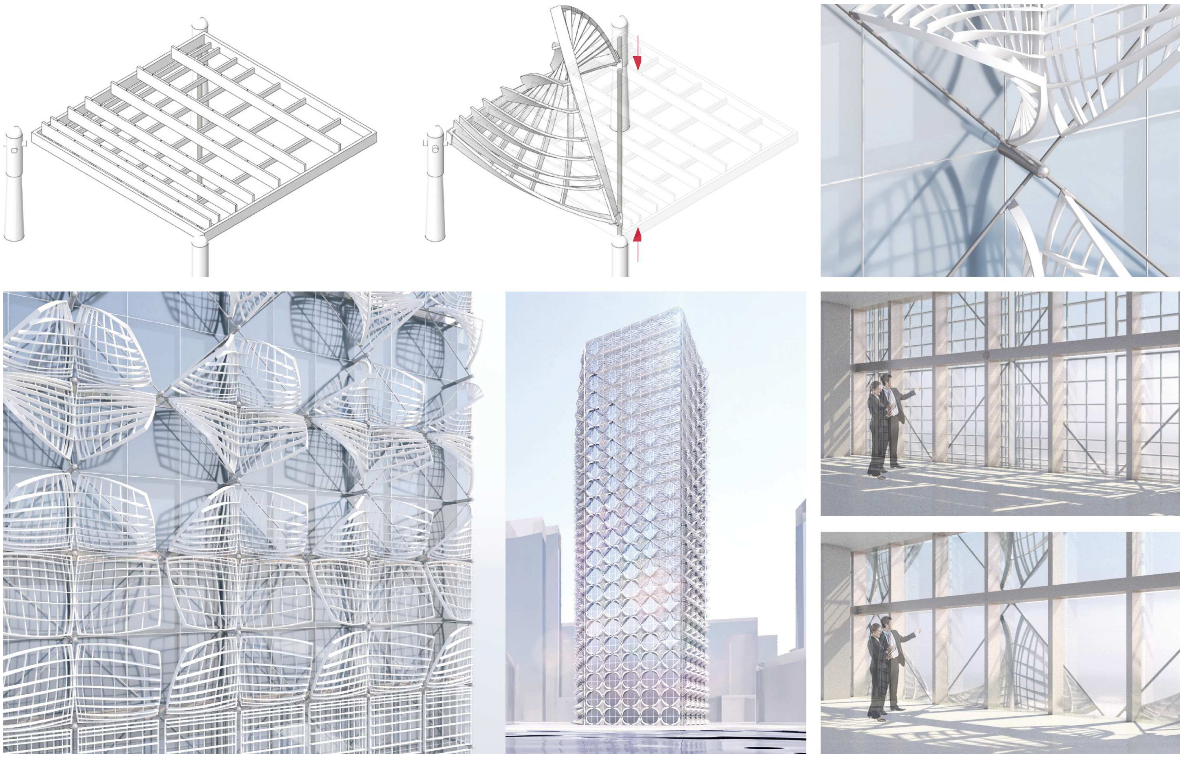

Figure 15: Transformable façade shading without hinges, using rectangular lamella grids. The compliant mechanism is actuated through a diagonal tension cable and transforms the flat grid into a curved blossom shape.

A digital research project looks at **elastic façade systems** for hinge-less automated shading (see the renders of **fig. 15**). In this scenario, the regular, square grid is fabricated from glass fibre reinforced plastic (GFRP) profiles. The modules can be installed on an existing façade. They are fixed at three corners and actuated through a diagonal tension cable. Pulling together the opposite corners creates a smooth transition from closed, flat panels with parallel louvers, to open, blossom-like shades, that can be adjusted to sun exposure. If the cables are released, the panels will naturally resume their initial minimum-energy state.

5.3 Kinetic Umbrella

The Kinetic Umbrella is a deployable structure, that is currently being developed (**fig. 16**). It will enter the construction phase in September 2020 and will hopefully be exhibited at the AAG2020 in Paris. The structure is designed from continuous GFRP profiles on two levels. For the lateral hinge connections centric bolts are used. The umbrella performs a semi-compliant transformation from a packed bundle to a deployed umbrella.

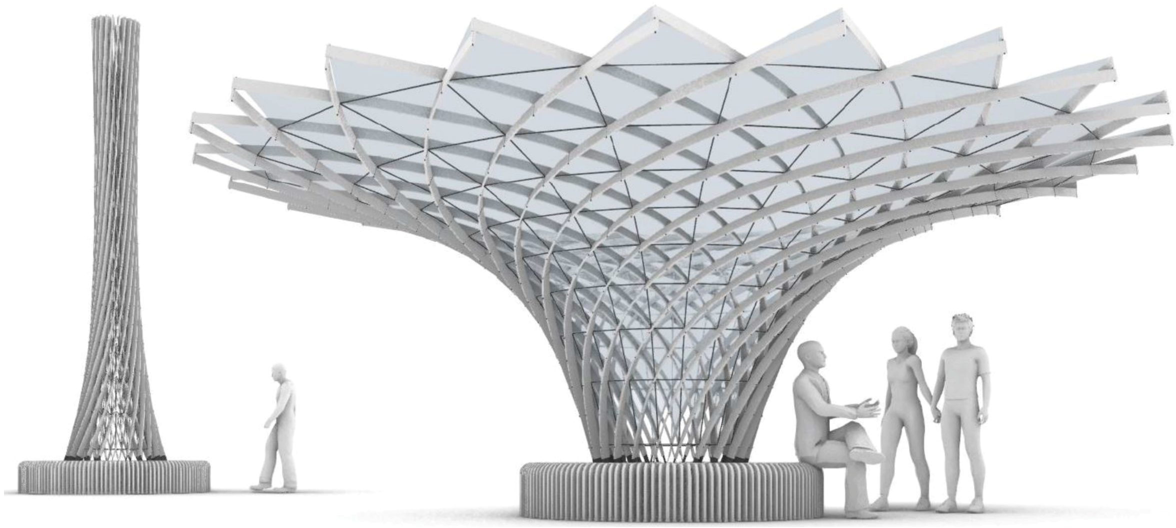

Figure 16: The Kinetic Umbrella. The rotational asymptotic network on an anticlastic surface (rotated NURBS) is constructed with two levels of GFRP hollow profiles. The profiles are slotted to create low torsional stiffness. They are connected with a centric bolts allowing for maximum rotation during the transformation process. The mechanism is actuated through ring and meridian cables.

Figure 17 gives an overview of the mechanical parameters and kinetic evaluation. First, we perform a simulation similar to **sec. 4.3**. In this case, the rotational grid will be actuated by forcing the upper diameter d to a range from $0.3m$ to $7.4m$ (**fig. 17** left). We record the curvature-square integrals for torsion and bending (**fig. 17**, top right). The contrary character can be recognised. We are now able to calculate the internal strain energy profile by multiplying the curvature-square graph with the corresponding stiffness parameters of GI_t and EI_z. By adjusting those stiffnesses, we may tune the graph and modify the kinetic behaviour. Even if the curvature-square due to torsion is higher than due to bending, the torsional energy becomes negligible, when slotting the rectangular section.

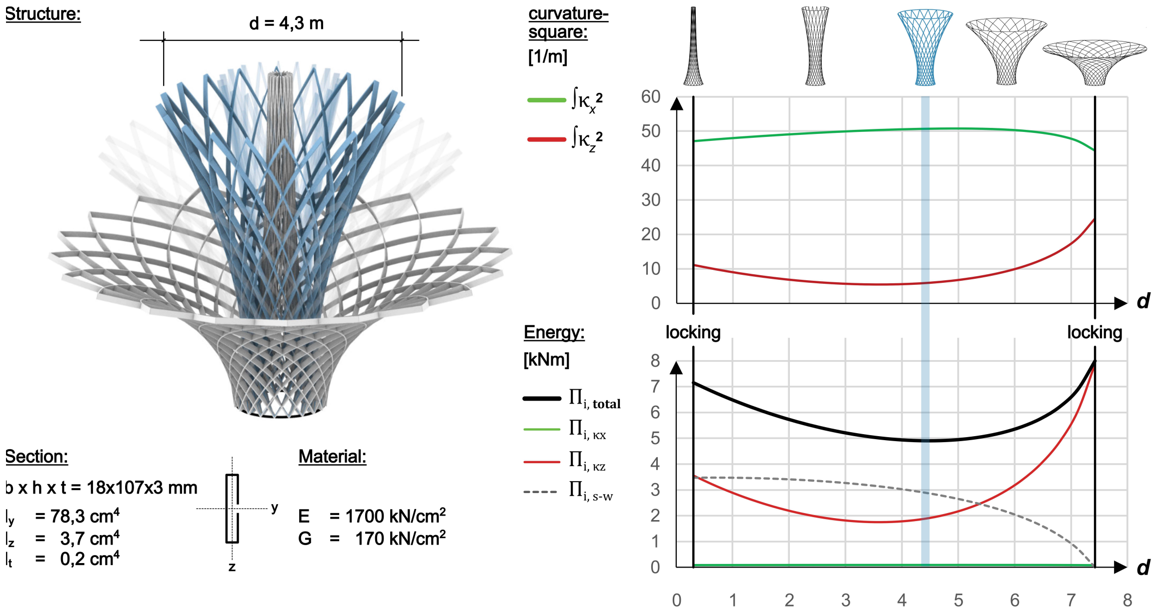

Figure 17: Structure, section, material values, curvature-square and energy profile of the Kinetic Umbrella.

Finally, the potential energy of the structure's mass is graphed with a dashed black line into the energy profile (**fig. 17**, bottom right). We sum up the energy portions and receive the full energy profile including self-weight (thick black line). The global energy minimum can be read out of the diagram at a diameter of $d = 4.47m$. We will need some actuation to both open and close the umbrella.

For further engineering, the deployed state is modelled and calculated using the FEM simulation method described in **sec. 3.2**. We apply real profile stiffness parameters, self-weight and eccentricities and receive a minimum state close to the predicted result derived from the curvature-square diagram ($d = 4.43m$). With additional cables that are used for actuation and locking, the deflection at open state is $10mm$. **Figure 18** shows the normal stresses for an open state utilising the material capacity up to 50%. The holes ($D = 8mm$) at each joint are not considered in the FEM analysis because their impact is insignificant (increase of max. normal stresses $+0.7\%$). The increase of shear stresses remains decisive.

Providing two layers allows full joint rotation but leads to additional bending moments acting on the lamella's strong axis, resulting from normal forces and eccentricities. These effects are not significant in this structure, as the profiles are mainly utilised by weak axis bending (see **fig. 18**). The rectangular sections are slotted to increase torsional flexibility. Alternatively, a double-layered section has similar mechanical properties and constructive qualities. Both allow central bolts for continuous profiles (see **fig. 19**).

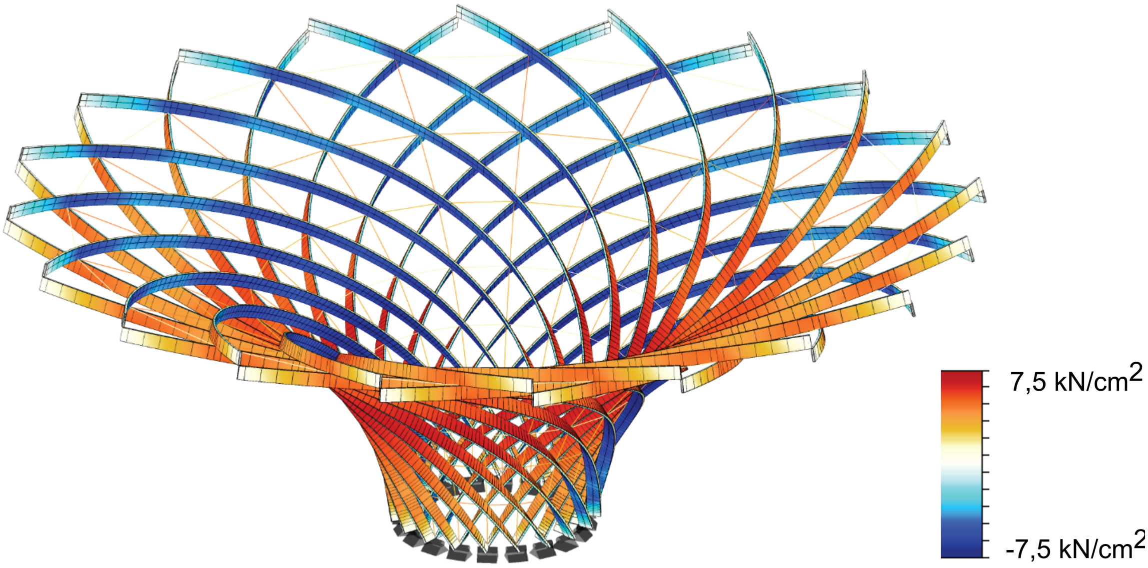

Figure 18: Stress analysis using FEM simulation method (see **sec. 3.2**) from deformed spatial geometry using Dlubal R-FEM – open state: stresses due to active bending and self-weight.

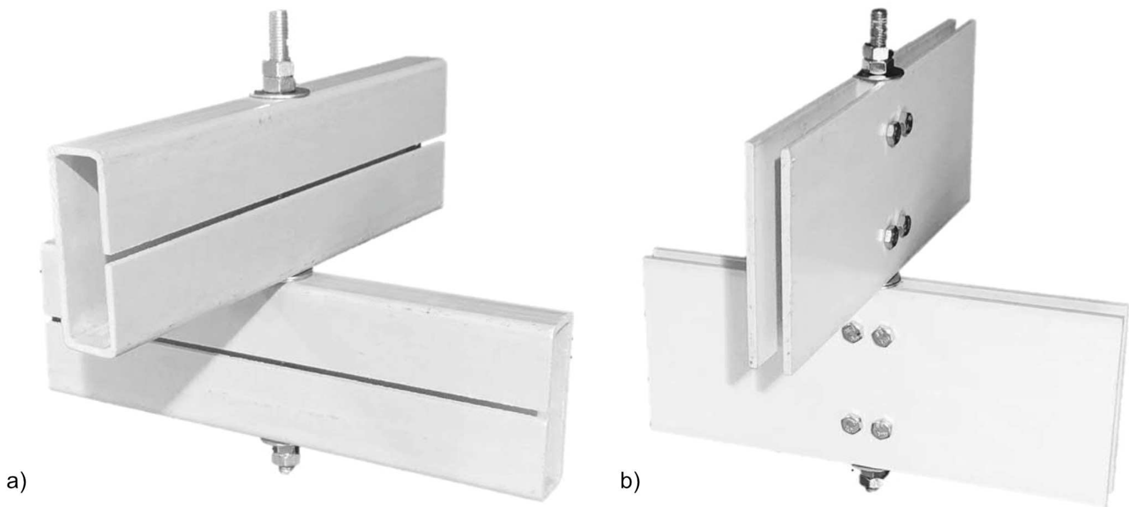

Figure 19: Prototypes of joints. a) The slotted rectangular section allows simple bolted connections. b) The double layered section is a suitable alternative (right).

The built structure is actuated through meridian and ring cables, which shorten the distance between joints and thus change the angles of rhombuses within the structure. Shortening the upper ring will close the umbrella, shortening the meridian cables will pull the umbrella down into its unfolded shape. Besides the profiles' bending capacities, the energy induced and stored is an issue of actuation and safety.

6 Conclusion

Quadrilateral grid structures hold great potentials to perform complex spatial transformations. Using beams with limited flexibility (torsion and/or uniaxial bending) and uniaxial scissor joints opens a new way of design thinking in the field of highly controllable semi-compliant mechanisms. We can categorise mechanisms on doubly ruled, geodesic, and asymptotic networks using appropriate profile sections. The flexural limitations lead to highly prescribed spatial deformations. We can characterise the kinetic behaviour using the curvature-square diagram, that represent the geometric factor of the structure's internal energy through transformation. It can be further used for practical engineering. Mechanisms on asymptotic networks potentially involve contrary action of bending and torsion. This property allows to control the structure's spatial state of equilibrium. At the same time, asymptotic gridshells show high potential for self-supporting gridshells resisting inextensional deformations.

Mechanisms on asymptotic networks were successfully used for the erection of static, elastic gridshells. The actual challenge is, to make such mechanisms accessible for transformable structures. This includes the analysis of the whole transformation process regarding relations between energy and shape. These relations become decisive for design and profile dimensioning. We present a variety of architectural applications ranging from adaptable façade to roof systems. A transformable umbrella structure is currently being developed, creating new insights into materials, construction, functionality, and aesthetics.

Future Research. The novel field of elastic grid mechanisms offers a multitude of further investigation, in both theory and practice. The numerical investigations discussed show the kinetic relations of basic shapes. We suppose there are clear derivations from differential geometry that may clarify and extend the range of shapes. We are eager to investigate the effect of helix torsion and extend the morphological studies of this typology, with dependency on beam, hinge and support parameters. Architectural practice will bring insights into the design workflow, fabrication and construction of joints and grids as well as stability, load-bearing behaviour, and façade solutions.

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