Curved-crease Paperfolding Structures and their Tectonics

Full scale experiments with mirror reflected cylinders

Duks Koschitz[1,*]

[1] Pratt Institute, Brooklyn NY
[*] Corresponding author e-mail: duks@pratt.edu

Abstract

Given the constraint of curved-crease paper folding, which type of sheet-structure with compliant hinges can be designed and fabricated? This paper investigates mirror reflections of developable surfaces for structures and provides the results of proof-of-concept prototypes that demonstrate how this specific topic in geometry can be used to create stiff surface structures. The specific method to create curved creases is based on mirror reflections of cylinders, cones and tangent surfaces, which can be constructed with CAD software. The goal of the presented case studies is to highlight design principles and tectonic solutions related to 2D manufacturing. This paper reports on projects such as a single surface shell, furniture designs, building parts and a double surface structure, all made of vulcanized fibre sheets. The material is made of polymerized cellulose and is available in large sheets. The material becomes malleable when it is wet, which enables the folding of compliant hinges. The findings of the built projects provide tectonic solutions that relate this topic in geometry to the logics of sheet assemblies.

Keywords: Curved-crease paper folding, Mirror reflections of developable surfaces, Compliant hinges.

1 Introduction

Curved creases have been explored by artists, designers and geometers from the 17th century in the form of table decorations to the Bauhaus (Demaine et al. 2011), where Albers used them in his foundation course. More recently geometers have further explored curved creases in a variety of ways that include constructive methods (Huffman 1976; Sergei and Fuchs 1999) and numerical approaches. However, there are few types of curved creases we can model directly in 3D software as there does not exist a general mathematical description of the behavior of a curved crease (Kilian et al. 2008). A small subset of this topic in geometry is well known and can be modelled in software, namely 'mirror reflections' of developable surfaces. The process requires designers to define a surface and to subsequently cut and reflect the surface with mirror planes.

The advantage of using curved creases in the building industry is related to its inherent energy benefits when creating 3D structures as no form work is needed. Architectural materials often come in the form of sheet goods, which are formed or assembled into architectural parts. Finding new concepts in geometry and relating them to building processes is relevant when the process requires little energy in the fabrication sequence. The basic premise of the presented case studies lies in using the geometric concept of curved creases and to realize small structures at full scale while using innovative solutions. The United Nations predicts that the existing global footprint of all cities will have to double by 2050 (UN DESA 2018), thus there is urgency in finding novel ways of building that are light and/ or fast. Using curved creases to create building parts results in light components as the surfaces are thin. If the surface structure follows force flows, the assembly can be extremely efficient. In this paper we are presenting the findings of three design research studios that were implemented in the fourth project. The final project is designed as double surface structure with curved creases that acts as a monocoque. This means that all surfaces are subjected to forces and that all parts of the envelope are integral to the main structural system.

1.1 Mirror reflections of developable surfaces

Paper, mathematically speaking, can only assume the shape of a developable surface between folded creases. A developable surface has to follow certain constraints, which makes it a special kind of ruled surface with a Gaussian curvature equal to 0. There are four main types of developable surfaces (Lawrence 2011): a planar surface, a generalized cylinder, a generalized cone and tangent surfaces. Any of these surfaces can be used in the proposed design process that consists of cutting an arbitrary developable surface with a plane and subsequently mirror reflecting the cut off part such that it fits on the intersection of the plane and initial surface (see fig. 1). This intersection can be described as a curved crease, which can be built as a compliant hinge meaning that the material is perforated and folded.

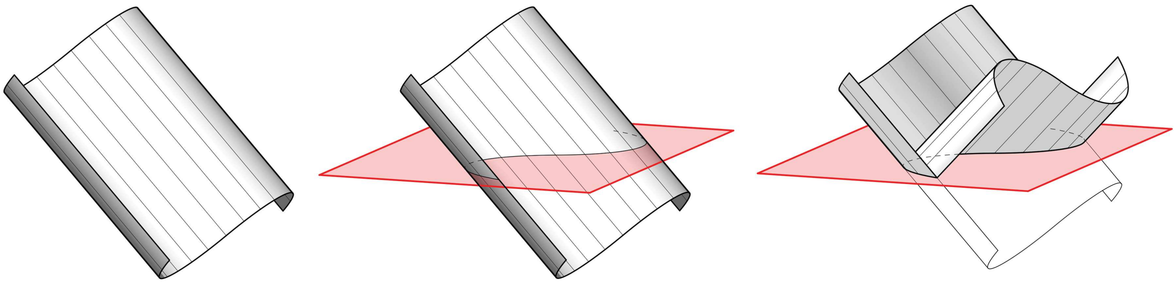

Figure 1: A general cylinder that is intersected by the mirror plane and its mirror reflection.

Jun Mitani has developed his ORI software (Mitani and Igarashi 2011) to demonstrate the versatility of the construction method for design. Most CAD software can be used to create this construction in a step-by-step process by manually cutting and mirroring the developable surface.

1.2 The four case studies and their methods

Over the course of the past three years I have conducted design research studios that operate on a trial and error basis. The constraints are well defined and a specific hypothesis guides each studio. Designs are tested at three scales in three different materials namely card stock, cardboard and vulcanized fibre board. All investigations use mirror reflections of cones and cylinders as their main design method. The four design research studios 'Single surface shell with ribs', 'Curved creases for furniture', 'Architectural components' and 'Curved creases and monocoques' were structured as a sequence in which one studio built on the previous findings and the accumulated findings are presented in this paper. The single surface shell was realized as a single sheet assembly that was first assembled flat on the ground and subsequently lifted in place on site. The furniture design studio focused on the refinement of mechanical fastening and tectonic ideas at a small scale. The subsequent study on architectural components resulted in advances in details for larger sub-assemblies that were designed as box-beams. Finally, the double surface structure explored the construction of a monocoque (see fig. 2).

1.3 An architectural precedent

Jean Prouve's Gas Station serves as architectural reference and inspiration (Prouve 2017) for the final structure presented in this paper. The building is prefabricated and the rear wall and roof assemblies are constructed in several bays. The manufacturing of a 2D building assembly that is formed into its final configuration in the form of a monocoque has structural and material efficiency benefits as all parts take on forces in the structural system. Prouvé was manipulating steel sheets and relied on stamping (Archieri et al. 2002) as a forming process, which is different to using surfaces with curved creases. However many principles of his details can be applied to sheet-based monocoques. His design also relied on primary and secondary structural systems, which we tried to merge in the final research studio. The column cover work by Haresh Lalvani should also be mentioned here as an important reference as he and Milgo Bufkin developed folding techniques for steel sheets that reached up to 4 m in length (Lalvani 2003).

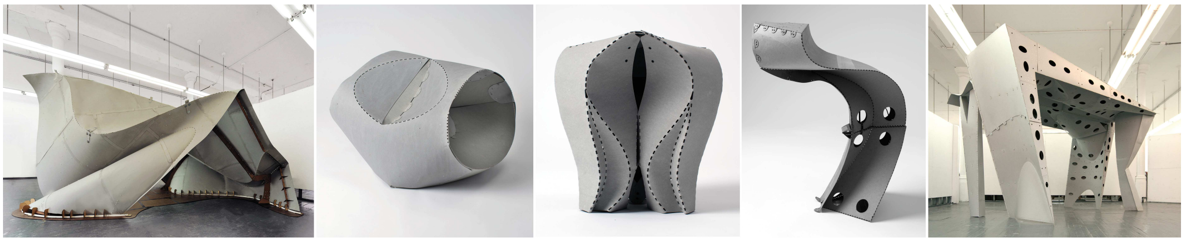

Figure 2: Examples of the four design research studios 'Single surface shell with ribs', 'Curved creases for furniture', 'Architectural components' and 'Curved creases and monocoques'.

2 An attempt to combine previous findings

The final study incorporates all findings of the previous projects and further develops sub-assembly to sub-assembly joints using a configuration made of two different bays (A and B). Each bay was designed with a moment connection on one side and a column on the other. The defined constraints included that it should be made as a monocoque, that access holes should only be located on the inner surface and that appropriate strategies for tolerance should be considered (see **fig. 3**).

2.1 The tectonics of sheet-based monocoques

In this section we discuss the process by which one can achieve a monocoque structure with curved creases. The development of details for vulcanized fiber relies on several of its material properties. The material is made of cellulose and is non-toxic, which has great benefits in a learning environment. It is, still today, unclear if the material is classified as plastic or paper as it is made of paper pulp subjected to an acid treatment that triggers a polymerization process. The contemporary use of the material is predominantly focused on electric isolation, but it was invented as an early 'plastic' material that was used for suitcases and engine gaskets. Regarding manufacturing processes it can be cut, laser cut and also milled, which is extremely useful during detail development phases.

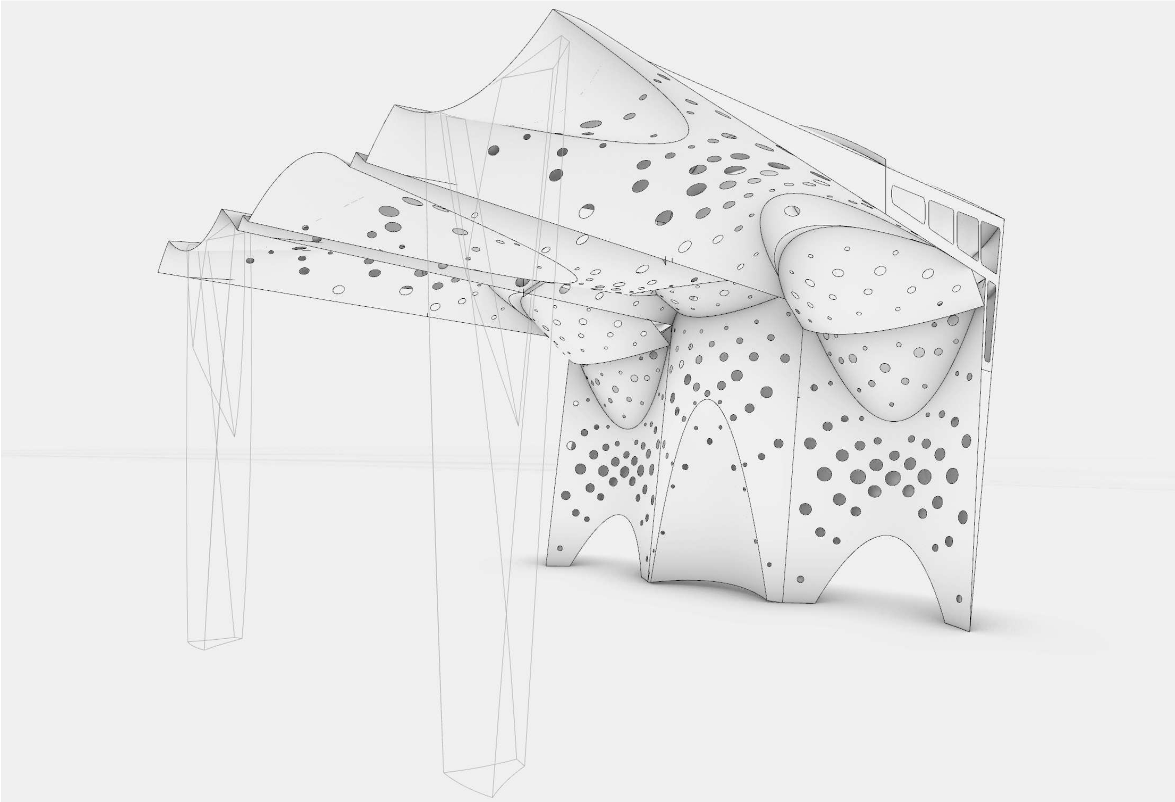

Figure 3: Diagram of final design, bay A in the center and bay B to its left and right.

2.2 Designing a double surface structure

When creating a monocoque with curved creases it is useful to explore design options where the outer and inner shell meet along a curved crease. This can provide a volume with two different sections (see fig. 4). The section along the rulings will consist of a polygon whereas the section through the cylindrical surfaces will be made of concatenated curve segments. The studies of the furniture studio aided in providing non-planar subassembly joints for the final structure.

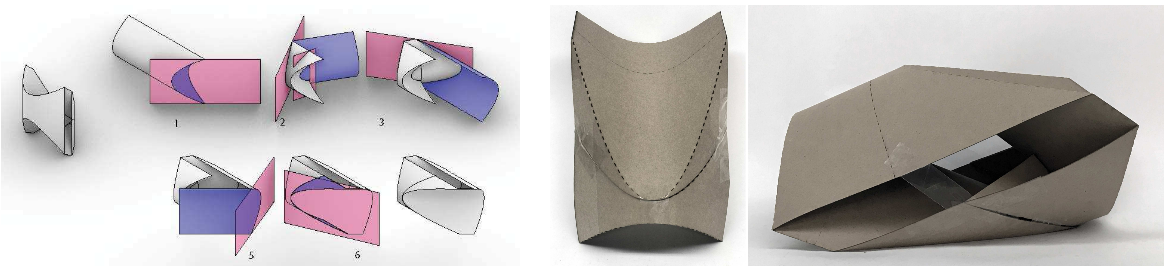

Figure 4: Diagram of volumetric study model, chipboard prototype.

Another exploration of an architecturally relevant detail relates to turning the corner, from a wall to a roof for example. The imposed constraint to create a moment connection meant that both, the outer and the inner surfaces had to turn from a horizontal to a vertical direction. A series of working models explored tuck folds, where two surfaces touch each other. The tuck can be seen in the area formed by the two degree-4 vertices in the crease pattern (see fig. 5 on the left). The study provided a way to turn the sheet form a vertical to a horizontal direction and the method was used in the final structure (see fig. 5 on the right).

Figure 5: Crease pattern and study model, front and side view of a chip board model of the final design.

The challenge to design a structure with two curved-crease surfaces lies in maintaining an appropriate distance between the surfaces. Following Prouve's design the distance between the surfaces reaches a local maximum in the transition from wall to roof (see fig. 7). The resulting section is designed such that the outer and inner surfaces meet at the foundation and at the tip of the cantilevering roof structure. The columns are also designed with curved creases and maintain a quadrilateral cross section throughout (see fig. 6).

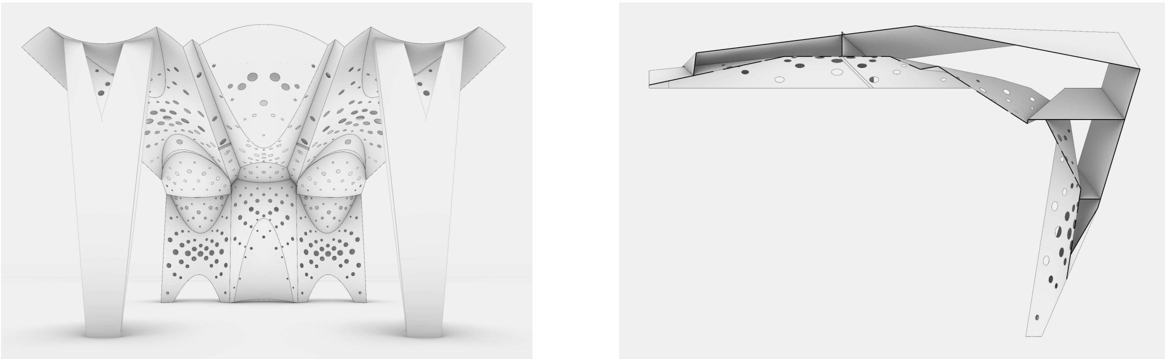

Figure 6: Image of final design with three bays, section through bay B.

2.3 Building sub-assemblies with curved creases

In order to improve the assembly sequence of the first single surface shell it was necessary to create sturdy components with creases shorter than 3 m. The studio on architectural components investigated the formation of box-beams with a triangular or trapezoidal cross section. The column is constructed in three parts or sub-assemblies (see fig. 8 on the left). The sub-assemblies meet along a plane constructed by parallel rulings, which allows for the installation of reinforcement

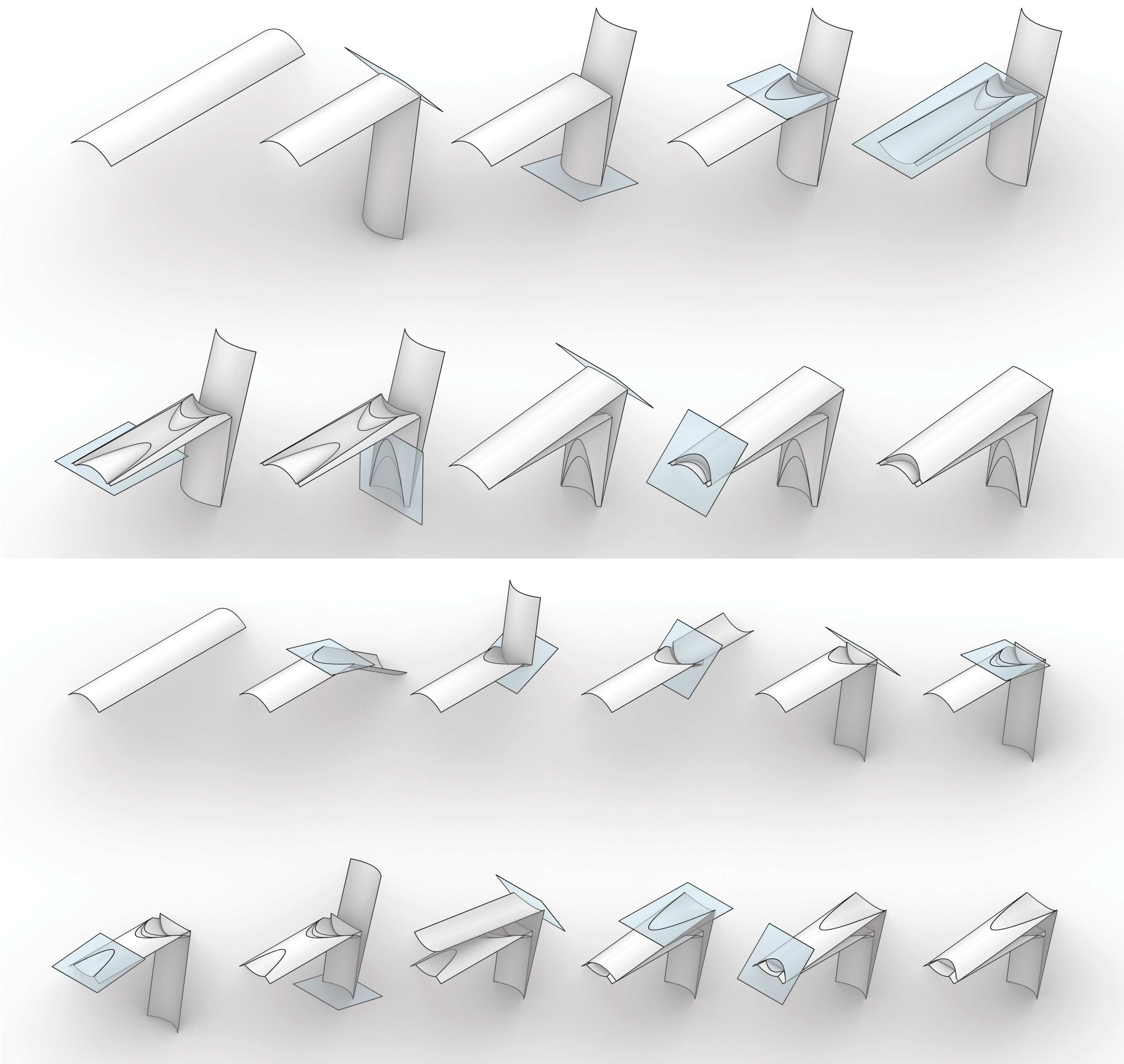

Figure 7: Diagram of mirror reflections to create both surfaces of bay A (above) and B (below).

plates. A 2.5 m long prototype of one sub-assembly proved to be difficult to construct as one of the seams of the box-beam was configured with two collinear mountain folds that were difficult to align. As a consequence major assembly joints between bays of the final project were designed to be flat and the mountain to mountain condition was avoided.

In order to further develop the principles of sub-assembly joints I imposed a design constraint that the joints for the final structure had to be made with curved creases. The stepped joints, made with lapped details, provided reinforcement in the areas where we designed bolted connections (see **fig. 9**). The curved surface segments provide useful guides from one sub-assembly to the next.

Bay A and B connect to each other along a plane, but the sub-assembly joints are made of cylindrical surfaces (see **fig. 10** on the left). The locations for bolts from one sub-assembly to the next are located in the joint or on the outer surface, which provides better force continuity (see **fig. 11**).

Figure 8: Three sub-assemblies of a column, planar joint, 2.5m large prototype.

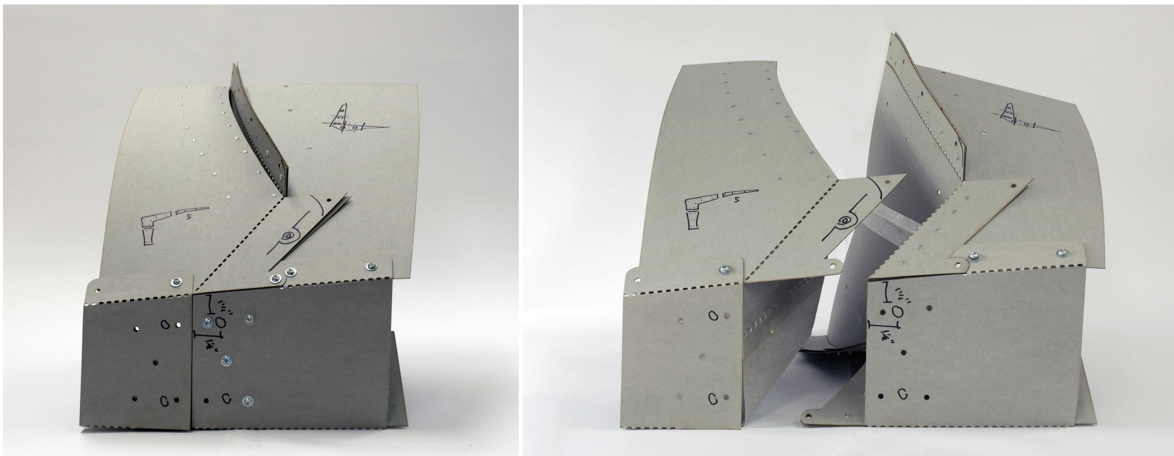

Figure 9: Prototype of sub-assembly joint; assembled position, open position

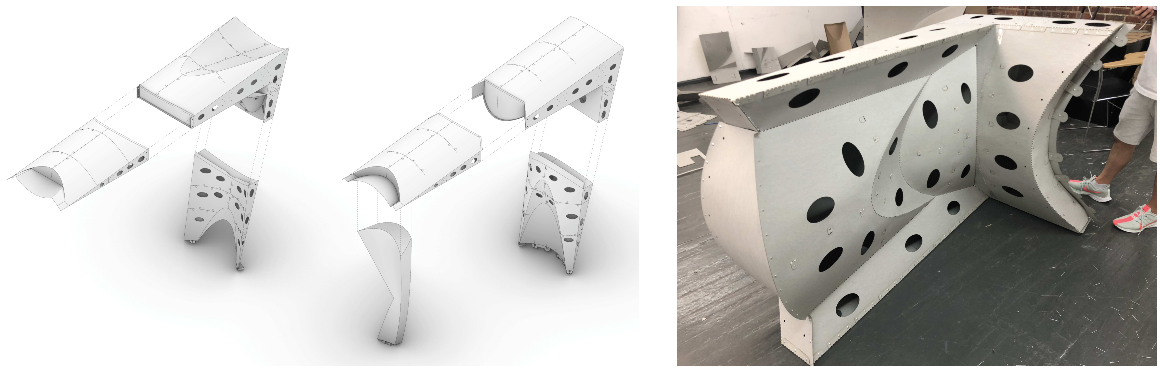

Figure 10: The 3 sub-assemblies per bay, corner sub-assembly of bay A with cylindrical surfaces at joint.

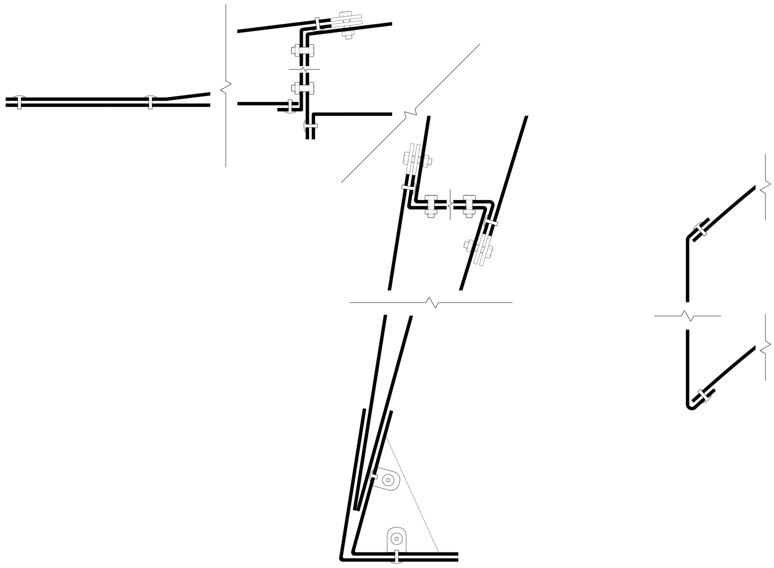

Figure 11: Longitudinal section and plan detail of bay A with permanent rivets and removable bolts.

2.4 CNC fabrication and sub-assembly construction

All fabrication is intended to be 2D and based on using complaint hinges throughout. Converging vertices, lapped joints and tolerances turned out to be the main concerns. Compliant hinges, the material continuity across a fold, can be achieved using perforations with material bridges of 25 mm for 1 mm and 1.5 mm material thicknesses. The seating design from a previous studio offered opportunities to study folding angles as it is made with a great variety of very shallow and very steep folding angles (see fig. 12). The material was folded up to 180° for some details, which can be achieved by using steam during the folding process.

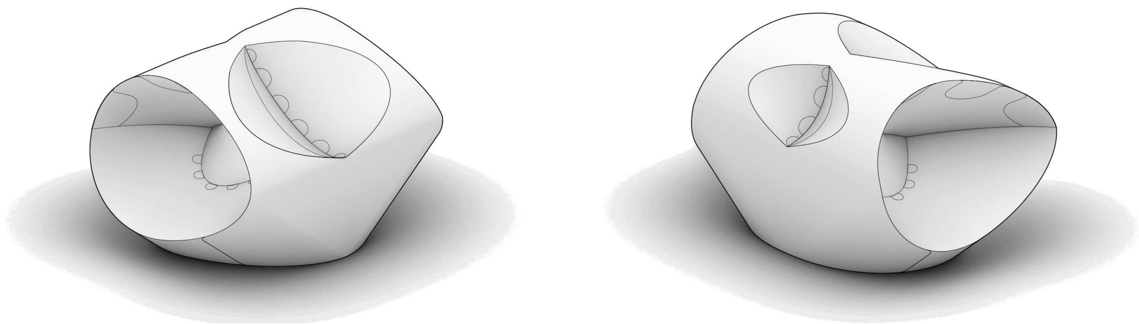

Figure 12: Design diagrams for seating design.

Wide material bridges for compliant hinges are strong whereas narrow material bridges are helpful for subtle folding angles. We however discovered that consistent material bridges of 25 mm combined with a variety of hole lengths can facilitate different folding angles (see fig. 13 on the left).

Sheet-to-sheet connections between two sheets can occur along the ruling, 90° to the ruling or at an arbitrary angle. Joints along the ruling and diagonally crossing the ruling proved to be difficult to assemble. Thus neither of these directions were used for the final structure. The use of aluminium rivets allowed for smooth surface continuity (see **fig. 13** on the right).

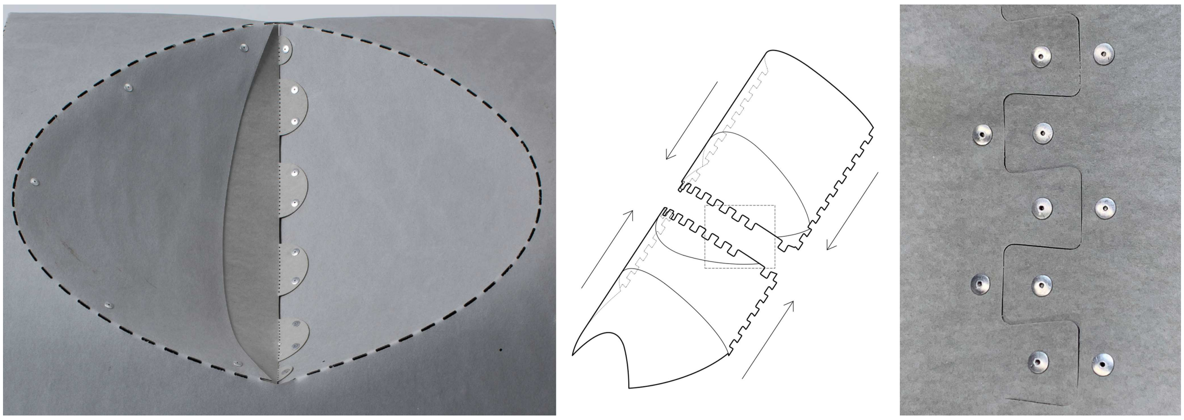

Figure 13: Prototype of seating with flaps and smooth sheet-to-sheet connection.

During the furniture design studio we encountered many converging creases and it became necessary to find appropriate details for degree 4-vertices (see **fig. 14**). The four compliant hinges come together in a way that can be challenging as the material needs to be folded in different directions within a very small area. Different variations of enlarged holes at the vertex proved to work reasonably well.

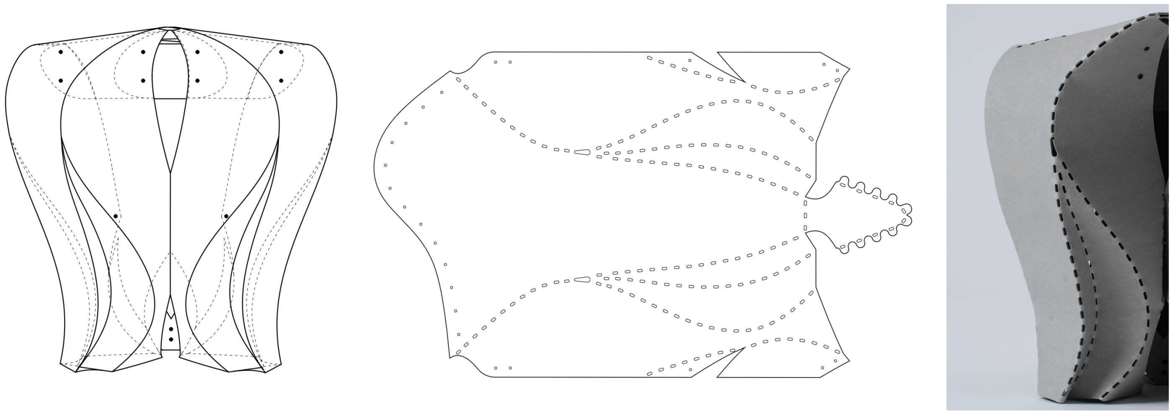

Figure 14: Elevation of stool design, CNC file that shows degree 4-vertex, detail of the vertex.

For the final structure it was necessary to test the location of creasing guides and stabilizing plates that ensure the proper position of every surface (see **fig. 15**). One assumption made for the final structure was that the stabilizers do not have to take on the main forces of a monocoque as the inner and outer surfaces are structural. As a result the locations of all stabilizers privilege requirements related to position control rather than force flow.

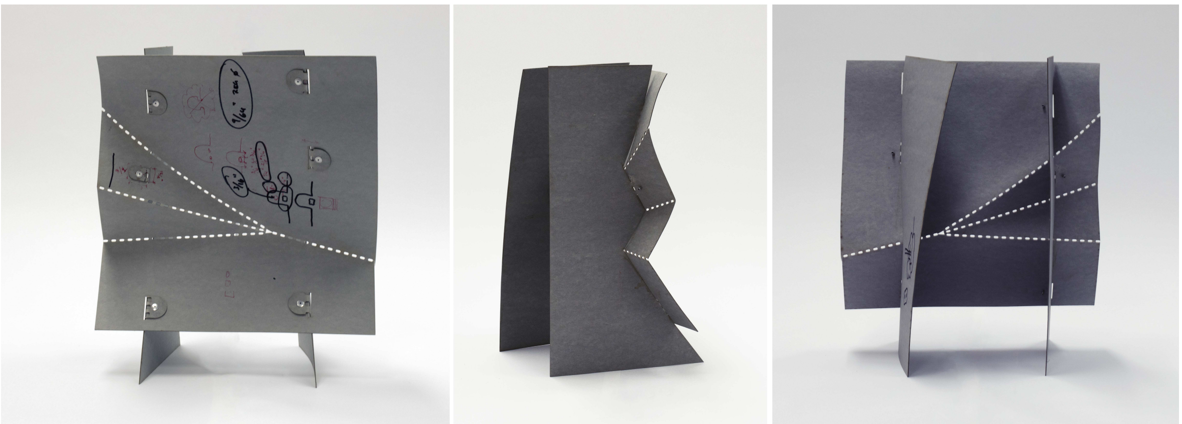

Figure 15: Prototype of final design with degree 4 vertex and 1 mm thick stabilizing plates .

The final design uses 1 mm surfaces for the inner and outer roof assembly, which are held in their correct position with 1.5 mm thick end caps. The Grasshopper definitions for the CNC files needed to accommodate a variety of sizes for tabs, slots and perforations (see fig. 16).

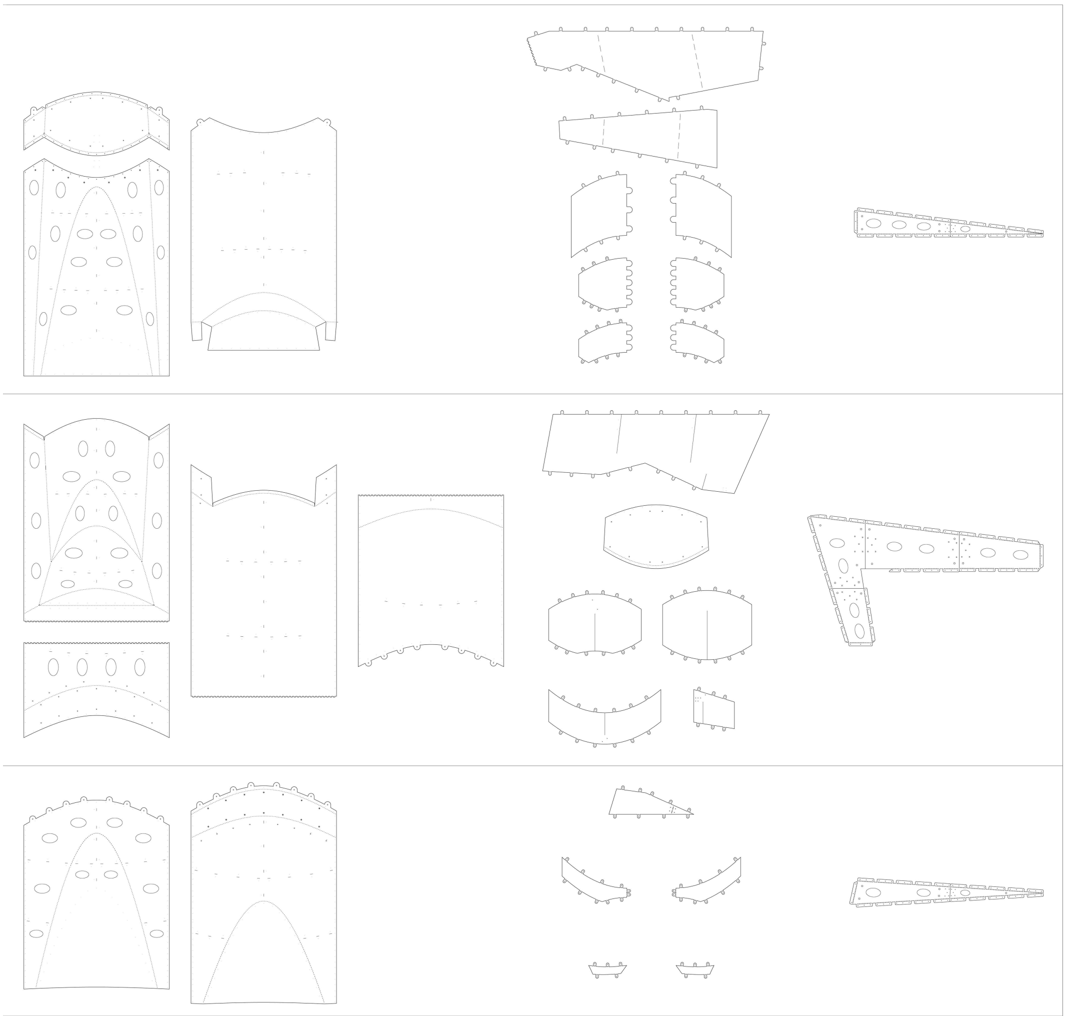

Figure 16: CNC cut files for bay A.

The three sub-assemblies were designed such that interior spacers ensure the correct position of both surfaces. The use of 'elastica' curves by Euler and Bernoulli proved very useful during the bending of the surfaces between creases. Elastica curves are based on the theory of bending bars or surfaces and which curvature they assume when they are bent. The resulting definition of curves applies to many materials in the realm of construction. The spacers are 1 mm thick, not continuous and are attached via tabs in oversized slots to provide tolerance. The assembly worked well as all assumed building sequences were adequate (see **fig. 17**).

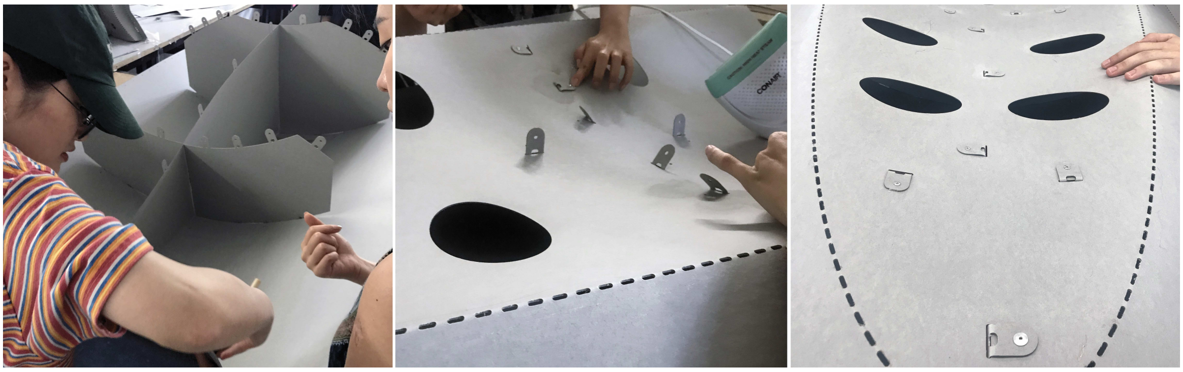

Figure 17: Surface spacers with tabs and installation of inner surface.

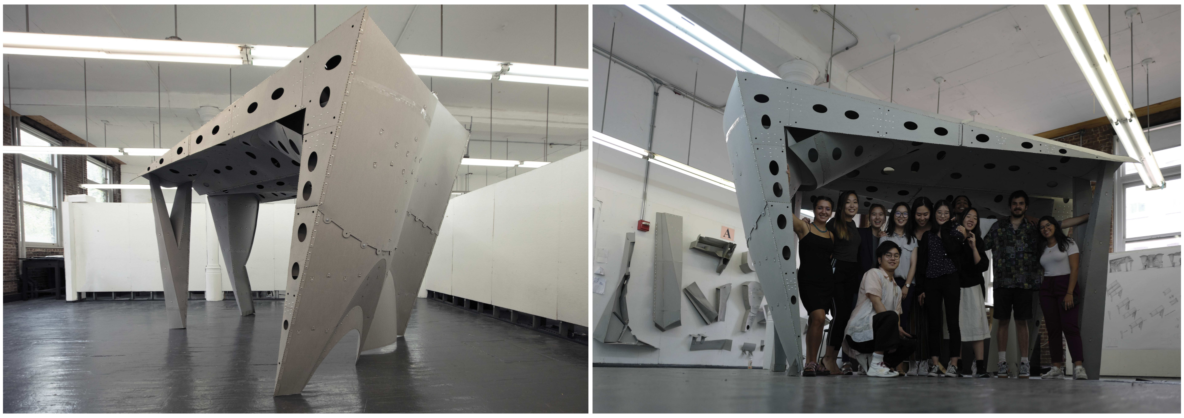

Figure 18: Final project and student team at final review.

2.5 The full scale prototype

The final structure consisted of bay A and B with two different column types, both with mostly quadrilateral cross sections (see **fig. 18** and **19**). The final assembly was reasonably fast and the assumed tolerance from one sub-assembly to the next worked well. The bolted connections required fewer bolts than planned and helped in maintaining surface continuity. The roof joint only needed 6 of 10 bolts to provide a perfectly level and strong connection. The limitation of providing access holes on the interior surfaces only did not pose any major issues regarding accessibility for tools and hardware. The column connections performed well.

Figure 19: Final structure.

The distribution of the sheets, 1 mm thick sheets for the roof structure and 1.5 mm thick sheets at the bottom of the walls, proved to be adequate. Minor deformations at the bottom of both bays should be addressed with an additional layer of material.

3 Summary of contributions and discussion

In the context of building assembly systems and construction methods, this work is proposing novel ways to manipulate sheet goods to create building components. Avoiding formwork or scaffolding has inherent energy efficiency benefits when compared to stamping metal parts for example. The findings of the four studies have been implemented in a final project that performs as a monocoque at the proof-of-concept level. The main ambition to correlate geometric concepts and their constraints to digital fabrication methods appears to be sound and useful.

The final structure was surprisingly stiff and the assembly was fast. A team of 6 students assembled the sub-assemblies and two bays over the course of two days. The applied tolerances were reasonable and all offsets for material thickness were adequate. The design, inspired by a specific work of Jean Prouve, led to the realization of a 90° wall to roof transition. Future explorations could investigate sections similar to vaulted shells. Next steps should also address the difficulties of working with stiffer and stronger materials such as polypropylene, composite panels or steel.

Acknowledgements

This series of design research projects was supported by a grant from the IDC Foundation in New York. The initiative is investigating novel, possibly faster construction systems that are making use of geometric concepts. Students learn how to work at full scale and are exposed to systematic design methods. The research assistants involved in several of the studies are: Xuechen Chen, Aisha AlJassim, Zaixing Huang and Angie Kim Park. The students involved in the research studios are: Sharon Broyn, Xuechen Chen, Chafiq Ennaoui, David Huh, Bharati Kodnani, Yen Chi Lee, Shaun Mehta, Jonathan Ovshayev, Massi Surratt, Yuheng Wu, Runyu Zheng, Anabel Baquerizo Carbo, Yimeng Sun, Lindsay Unger, Aisha AlJassim, Pintian Liu, Eva Bajko, Joshua Cooper, Uarnice Hong, Zaixing Huang, Chongsoo Jeon, Angie Kim Park, SoYeon Lee, Yuanming Ma, Safa Mehrjui Seyed, Maria Rosario Flete, Mingyang Sun, Taline Tenguerian.

References

Archieri, J.-F., J.-P. Levasseur, and H. Damisch (2002). *Prouve: Cours Du CNAM, 1957-1970*. Sprimont, Belgium: Mardaga.

Demaine, E. D., M. L. Demaine, D. Koschitz, and T. Tachi (2011). Curved crease folding: A review on art, design and mathematics. In *IASS Symposium*.

Huffman, D. (1976). Curvature and creases: A primer on paper. *IEEE Transactions on Computers 25*(10), 1010–19.

Kilian, M., S. Flöry, Z. Chen, N. J. Mitra, A. Sheffer, and H. Pottmann (2008). Curved folding. In *ACM SIGGRAPH 2008*.

Lalvani, H. (2003). Algorhythm technologies.

Lawrence, S. (2011). Developable surfaces: Their history and application. *Nexus Network Journal 13*(3), 701–14.

Mitani, J. and T. Igarashi (2011). Interactive design of planar curved folding by reflection. In *Pacific Graphics*.

Prouve, J. (2017). *Bilingual edition*. Galerie Patrick Seguin.

Sergei, T. and D. Fuchs (1999). More on paperfolding. *The American Mathematical Monthly* (106), 27–35.

UN DESA, U. (2018). Around 2.5 billion more people will be living in cities by 2050, projects new un report.