Meshing with Kagome Singularities

Topology adjustment for representing weaves with double curvature

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Abstract

In this paper, we present a planar mesh topology adjustment scheme for the automated production of principled Kagome weave patterns.

Kagome is a triaxial weave system based on a hexagonal lattice. Double curvature is induced by replacing hexagonal cells with alternative polygons. These singularities are digitally represented by constructing a tri-mesh with appropriate valence. However, the automated adjustment of meshes remains an open challenge.

The topology adjustment scheme presented here employs edge flipping to modify a regular valence 6 mesh. A key insight is that embedding a local valence modification necessitates a cascade of edge flips that extend out to the mesh boundary.

We demonstrate the application of this principle to represent a range of singularity valences and verify these individually with simulation. A series of case studies demonstrate how: 1) the scheme can accommodate multiple singularities within a mesh; 2) the scheme can be integrated into a digital design pipeline that includes fabrication documentation production for producing physically woven Kagome approximations using only straight strips of material.

Within the paper, we contextualise this work with reference to the state-of-the-art, articulate its contribution, discuss the limits of the scheme and reflect upon its broader relevance to architectural design and construction.

Keywords: Kagome, weaving, mesh topology, mesh valence, topology adjustment, edge flipping, double curvature, digital design, simulation.

1 Introduction

The study of Kagome weaving has seen recent advancement from within, and with direct application to, the fields of architecture and engineering. These research efforts can be distinguished into two focus areas; 1) representation and instrumentation of underlying principles in the form of digital methods and tools to support exploration, specification and analyse performance (Mallos 2009; Pottmann et al. 2010; Deng et al. 2011; Mesnil et al. 2017; Ayres et al. 2018); 2) fabrication approaches and construction concepts seeking to address architectural scales whilst preserving weaving principles (Huang et al. 2017; Ayres et al. 2019, 2020). The work presented here is motivated by the fact that the recent approaches to weave representation, as reported in the literature, lack methods for automated topology adjustment of the design mesh. This presents a bottleneck within the designer's workflow and therefore acts as an impediment to rapid design iteration and analysis.

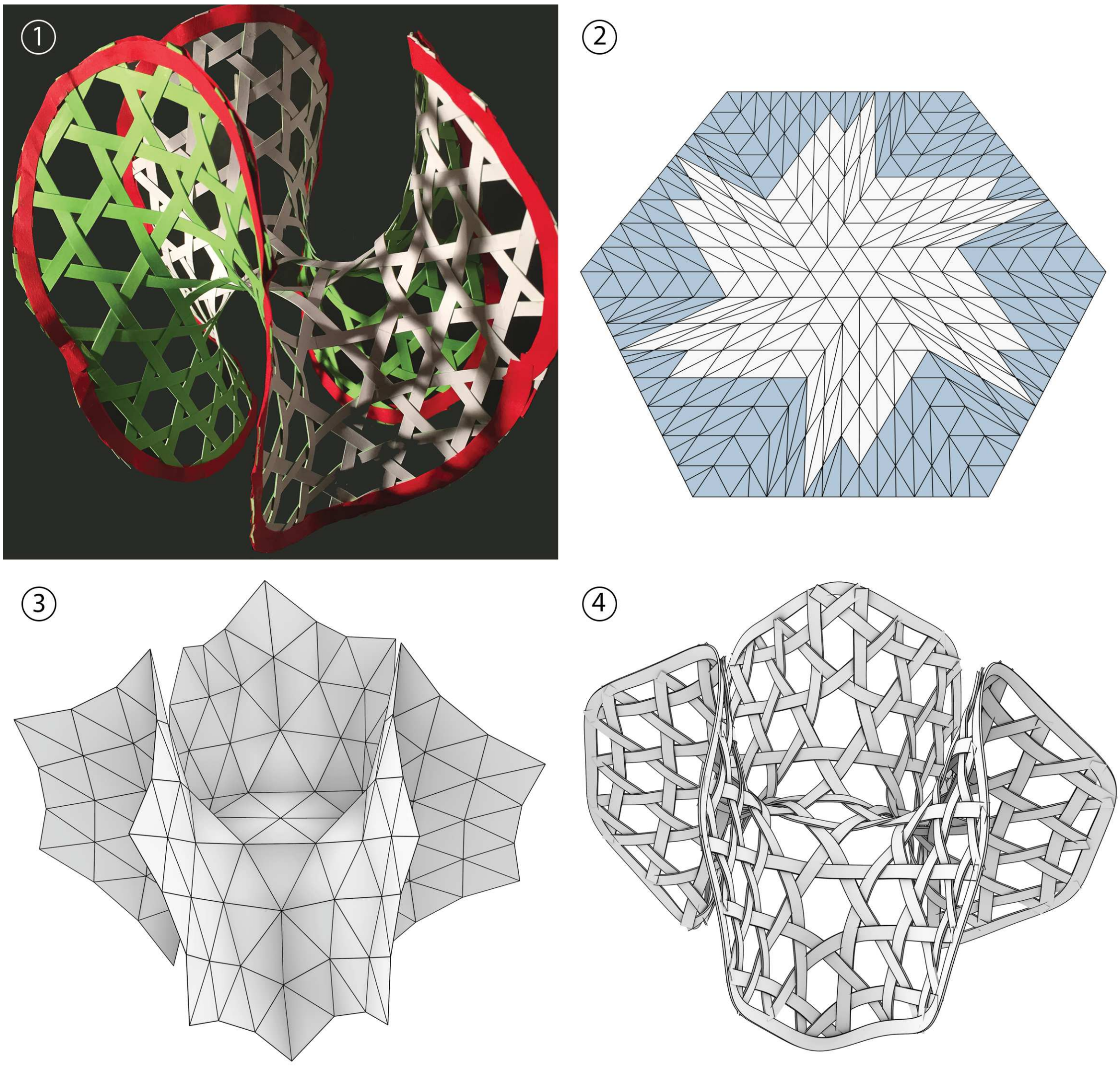

Figure 1: Steps in the modelling of a Kagome weave: 1. the hyperbolic physical weave to be represented; 2. topologically adjusted mesh using the Mesh Topology Adjustment Scheme (MTAS) presented in this paper; 3. relaxed mesh; 4. interlaced weave representation.

In this paper, we focus on the study area of representation and target this open challenge of real-time mesh topology adjustment in order to facilitate design exploration (**fig. 1**). Here, the term singularity refers to the introduction of a non-hexagonal polygon cell into a physical weave in order to generate local double curvature when using straight strips or filaments of material. Lower sided polygons (i.e < 6) induce synclastic curvature, with more pronounced curvature the lower the number of sides. Higher sided polygons (i.e > 6) induce anticlastic curvature, with greater curvature generated as the number of sides increases. Following these principles, the representation of singularities in regular tri-meshes is accomplished by adjusting local vertex valence to be equivalent to the number of sides of the intended replacement polygon.

Using the basic mesh topology operation of 'edge flipping', we demonstrate mesh topology adjustments to regular planar valence 6 meshes for representing principled Kagome weave patterns across the valence set $[v|3 \leq v \leq 9]$. A key insight of the work is that local valence modification implicates a series of associated edge flips for specific regions of mesh faces, running out to the mesh boundary. This is necessary in order to maintain $v6$ regularity around the singularity.

In the following section, we briefly cover the state-of-the-art in Kagome weave representation methods and identify mesh topology adjustment methods relevant for the task being addressed here. In **sec. 3**, we describe the methods underlying our mesh topology adjustment scheme (MTAS) and demonstrate how it is employed to represent a set of singularities for the domain defined above. Two case studies are then presented that explore the use of the scheme for producing meshes with multiple singularities. A further case study examines the embedding of the MTAS into a simple digital design pipeline that allows direct, interactive specification of vertex sites and valence, together with providing fabrication information for physically realising the weave. In **sec. 4**, we reflect upon the results of the case-studies to draw out the merits and limits of the MTAS. In **sec. 5**, we conclude by articulating the contribution of the work presented in relation to the state-of-the-art and offer vectors for further work.

2 Background

According to Semper (1851), architectural enclosure finds its origin in the outcomes of weaving, with textiles performing the roles of spatial dividers and surface coverings in the form of screens and mats. Weaving continues to have architectural relevance today both as a spatial concept (figuratively and literally) and as a practical production method. Advances in materials, methods of production and methods of representation have broadened the scope of architectural usage and continue to drive novel applications (Born et al. 2016; Heinrich et al. 2016), novel digital generation approaches (Nejur and Steinfeld 2017) and spur wider agendas of architectural research enquiry spanning notions such as Spueybroek's *Textile tectonics* (Ludovica Tramontin 2006) - which tends towards formal and figurative outcomes - and *Textile logics* (Ramsgaard Thomsen and Bech 2011; Thomsen et al. 2012) which focuses on the role of digital representation and simulation in the study and transfer of textile organisations and principles to architectural scale through the optic of material performance.

The work presented here contributes to the latter agenda, seeking to address an open challenge in the representation of Kagome patterns for the purpose of supporting the transfer of principles and techniques to be applied at architectural scale. In the following section, we briefly survey the state-of-the-art of representational approaches specifically targeting Kagome patterns.

2.1 State-of-the-art: existing Kagome representation methods

Computational approaches for the representation of Kagome patterns have been studied and reported in the literature, but with limits and open challenges still remaining for application within the context of architectural design. Here, we briefly examine a selection of these and evaluate them against two compatibility tests for architectural application; that:

1. the approach supports the generation of weave patterns made from straight strips of material thereby rationalising production, saving material and placing the intelligence for achieving geometric approximations of design targets into the interaction between diligent topological design and material performance;
2. the approach provides design agency in the form of interactive editing of mesh topologies and provides real-time feedback of the consequences on morphology.

The medial construction method described by Mallos (2009), provides an efficient and elegant means of obtaining a Kagome pattern from any manifold tri-mesh by drawing the in-triangle of every mesh face. However, the limitation of the approach is that singularity positions do not necessarily conform to the principles for achieving double curvature. The consequence is that the method does not satisfy the first test of generating weave patterns that can be realised using straight strips.

In Huang et al. (2017), a mesh is generated from a predetermined input surface. A remeshing algorithm is used to refine the mesh, optimising edge lengths to suit mechanical and dimension properties of the weaver material. However, it is unclear how valence of the mesh is controlled through this optimisation as iterative edge splitting and collapsing operations appear to either retain $v4$, or introduce $v4$ conditions. Produced weaves are demonstrated and constructed from FRP rods and PC pipes, thereby allowing deviations from the weaver straightness constraint. Weavers are very sparsely interlaced to allow large bending deformations in the weavers to approximate the target geometry.

In Ayres et al. (2018), the medial construction method of Mallos (2009) is extended with a mesh relaxation procedure to satisfy the straight strip constraint. However, design meshes are predetermined and manually constructed which is a hindrance to fluid and flexible design investigation, therefore not satisfying the second test.

2.2 Meshes and remeshing

The process of remeshing is a common and often necessary precursor to the effective use of meshes in a broad range of tasks, across many fields. Remeshing seeks to improve or adjust attributes of an input mesh to better suit the requirements or constraints of the target task - which can generally be classified as being either a topological transformation task or a smoothing task. In their survey paper, Alliez et al. (2008) offer a five category classification of the numerous remeshing techniques reported in the literature - structured, compatible, high quality, feature and error-driven. Common to many of the topology adjusting algorithms is the use of three basic mesh operators - flip, collapse, split.

In many structured and high quality techniques the task is to reduce irregular valences to create a more regular mesh. This is in contrast to our stated aim of introducing specific irregularities at defined locations. An interesting exception is the high quality remeshing technique of Turk (1992) which employs an attraction-repulsion particle relaxation method operating on user-defined input vertex locations. However, the remeshing is not aimed at producing principled Kagome patterns. Feature based algorithms tend towards feature preservation rather than feature introduction, which is the focus of the task investigated here. Compatible remeshing focuses on new mesh generation from correspondences between sets of input meshes; error-driven techniques are predicated on having a defined design target. Both classes of approach are not suited to the task being investigated here.

In summary, the task defined in this paper is to introduce local valence changes at specific vertex locations whilst preserving regular $v6$ topology across all other vertices, with the exception of boundary, or ‘naked’, vertices. To the best of the authors knowledge, there exists a gap in digital representation methods for automated topology adjustment in regular $v6$ planar tri-meshes to specifically produce doubly-curved weave patterns that conform to Kagome principles and that are producible from straight strips of material.

3 Methods

In Kagome hand weaving, singularities are introduced into the developing boundary of the weave by either omitting weavers, in $< v6$ cases for inducing synclastic curvature, or adding weavers, in $> v6$ cases for inducing anticlastic curvature. Weavers are linear members that contact multiple polygons on their route through the weave, therefore, the implication of local valence adjustment is that there is a global topological consequence on the weave pattern.

Embedding a local valence adjustment within a regular mesh has an analogous global topological consequence. Let us define the task of embedding a local $v5$ singularity within a regular $v6$ mesh at vertex location Vs. Vs must belong to the set of non-naked vertices - i.e it cannot lie on the mesh boundary. Using the basic topological mesh operator of edge flipping, we can locally alter the valence of Vs to $v5$ with one edge flip. However, this results in three collateral and unwanted valence adjustments, turning valence counts from 6-6-6-6 to 5-7-7-5 (fig. 2). To compensate, the unwanted singularities are altered back to $v6$ by applying a flip to an edge parallel to the original and moving away from Vs (fig. 3). This process is iterated, effectively 'chasing' collateral valence changes until they are 'flipped out' at the mesh boundary. This defines the general valence adjustment principle which results in a region of mesh faces with flipped edges and a local singularity embedded within a $v6$ mesh.

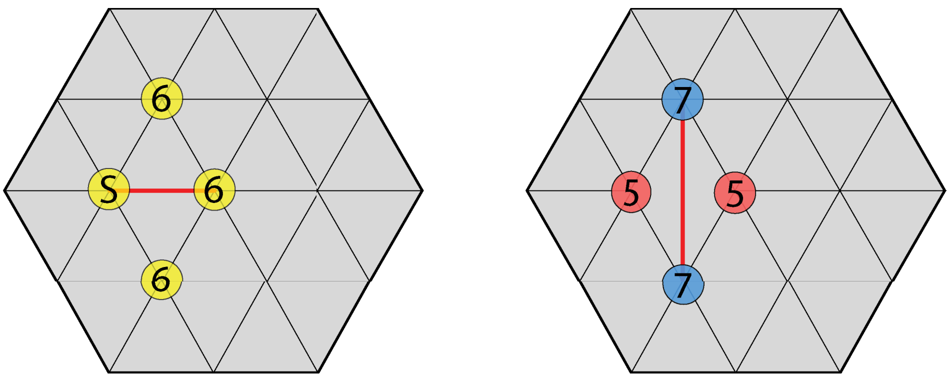

Figure 2: An edge flip produces valence changes on four vertices.

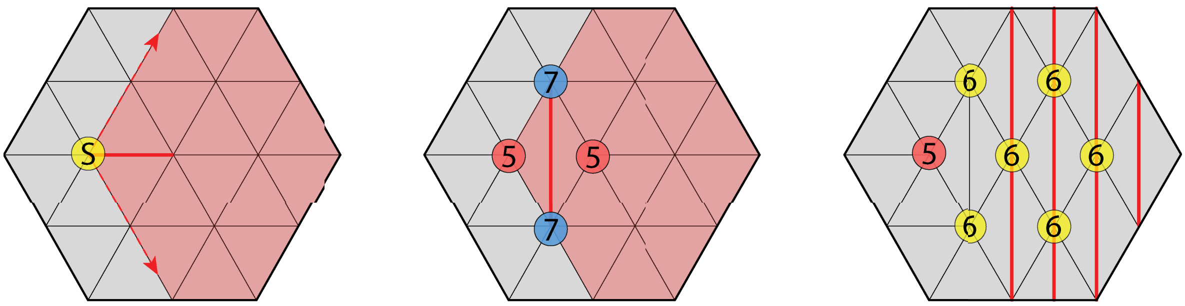

Figure 3: A flip-region for a $v5$ singularity.

3.1 Flipping out - half-edges, burning fronts and boundary conditions

The flipping out principle described above is implemented using a half-edge data structure which represents meshes as sets of vertices, edges and faces together with information on their connectivity. Vs is surrounded by twelve possible edges that can be flipped to adjust valence. Six of these are directly connected and the remainder related by the connected face. To lower the valence to $v5$ (for synclastic deformation), an edge directly connected to Vs is selected for flipping; to increase the valence to $v7$ (for anticlastic deformation), the unconnected edge of a face connected to Vs is selected (fig. 4).

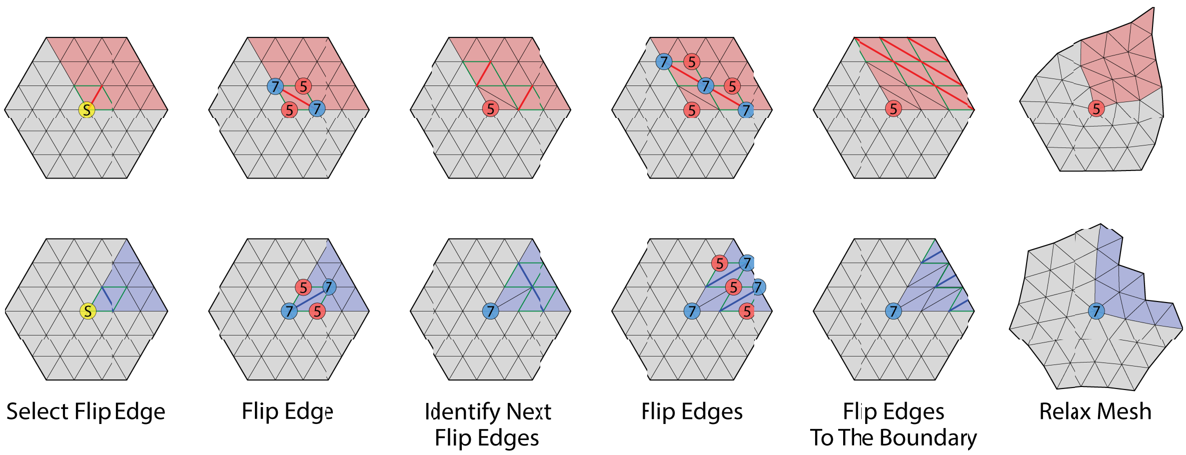

Figure 4: 'Burning fronts' for $v5$ and $v7$. Note the change in orientation of the flipped edges. The coloured areas indicate the extent of the flipping region.

To resolve the three collateral valence changes ($7-5-7$ and $5-7-5$ respectively) new edges to be flipped are identified through the half-edge paired face relationships. This process is iterated creating a 'burning front' until the unwanted valences changes have been chased out to the mesh boundary and leaving a singularity surrounded by $v6$'s.

At the boundary, vertex valence in a regular $v6$ mesh is naturally < 6. When flipping edges for a $v7$ singularity, this causes issues in maintaining $v6$ conditions at the first layer of non-naked vertices. To compensate, naked edges within a flip-region are split and a new edge added between the new vertex and its associated internal vertex (**fig. 5**).

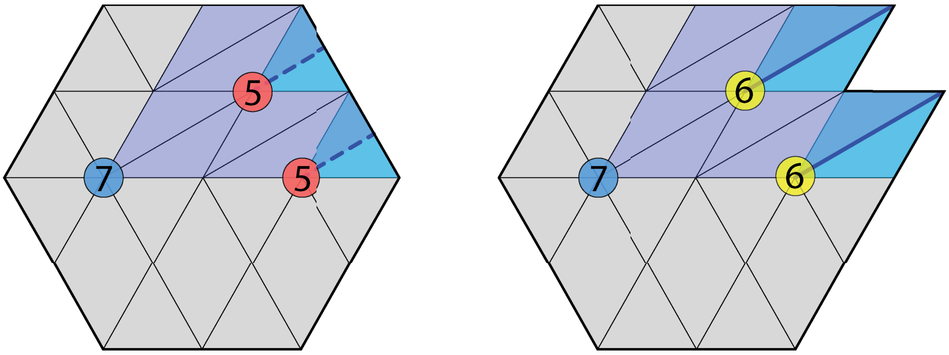

Figure 5: Boundary conditions can cause irregular internal valences to be introduced when edge flipping. This is resolved by splitting naked edges to maintain regular valence on internal vertices.

3.2 The singularity zoo

The scheme for introducing $v5$ and $v7$ singularities described in the previous section can be extended to represent the complete target set $[v|3 \leq v \leq 9]$.

In the cases $v4$ and $v8$, two edges must be flipped and in the cases $v3$ and $v9$, three edges. **Figure 6a** illustrates the various valence adjustments ($v3$ far left through to $v9$ far right) to a regular planar $v6$. In the top row, the respective flip-regions for each of the singularities are shown emanating from the same Vs located in the centre of the regular mesh. Planar relaxations are shown in the second row and out-of-plane relaxations in the third row. The fourth row shows the mesh converted to the interlaced Kagome pattern. The increasingly pronounced curvatures of the Kagome weave as valence moves to the extremes, correspond to the out-of-plane deformations occurring in physical weaves as shown in **fig. 6b**.

Of note, is the effect on curvature distribution and boundary geometry resulting from possible variations of flip-region adjacencies in the cases $v4$, $v8$ and $v9$. In the cases illustrated in **fig. 6**, the flip-regions are distributed without direct adjacencies to each other. Direct neighbourships of flip-regions are both feasible and legal within the MTAS, but the effects on relaxation outcome are subtly different as shown for the various combinations for producing a $v8$ shown in **fig. 7**.

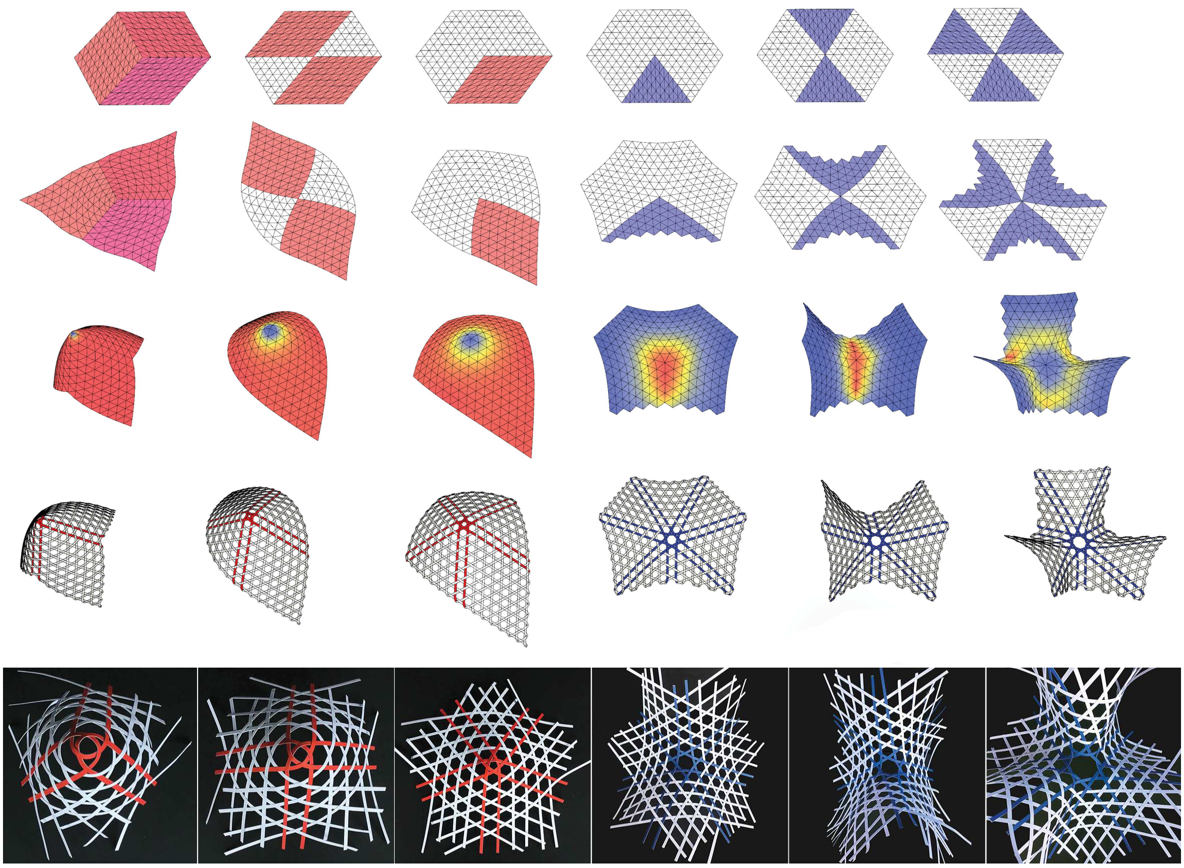

Figure 6: The singularity zoo. (**a**) All members of the target valence set can be achieved using the region flipping principle. (**b**) Bottom row shows physical equivalents to the members of the singularity zoo and illustrates the close approximation in double curvature.

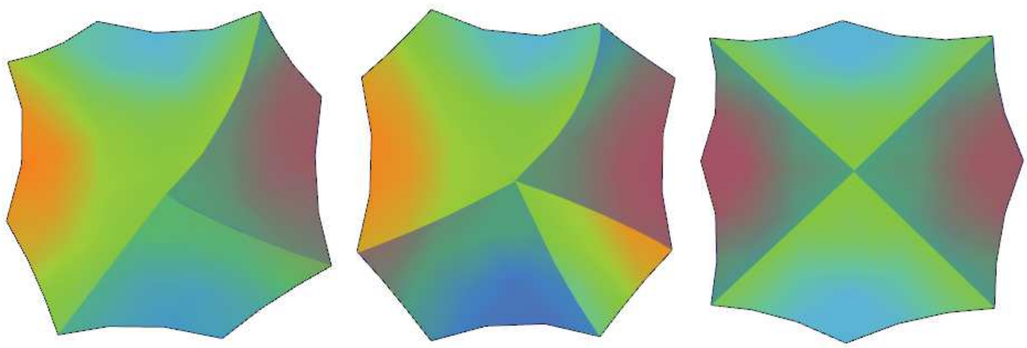

Figure 7: Analysis of the curvature effects resulting from different flip-region adjacencies.

3.3 Case study 1 – multiple singularities with no region crossings

This case study examines the introduction of multiple singularities into the regular mesh, but with the condition that flip-regions do not cross. In simple cases, all singularities can be introduced and relaxed simultaneously as there are no compound relationships - i.e mesh edges that require flipping will only do so once. However, the non-crossing condition places limits on the number and location of singularities that can be introduced. These limits are a product of the underlying topology of the regular mesh and the choice of valence adjustments. Variations to illustrate these are shown in **fig. 8**. Nesting within flipped regions can be used as a strategy to increase the number of singularity sites, but this requires careful sequencing of the edge flips.

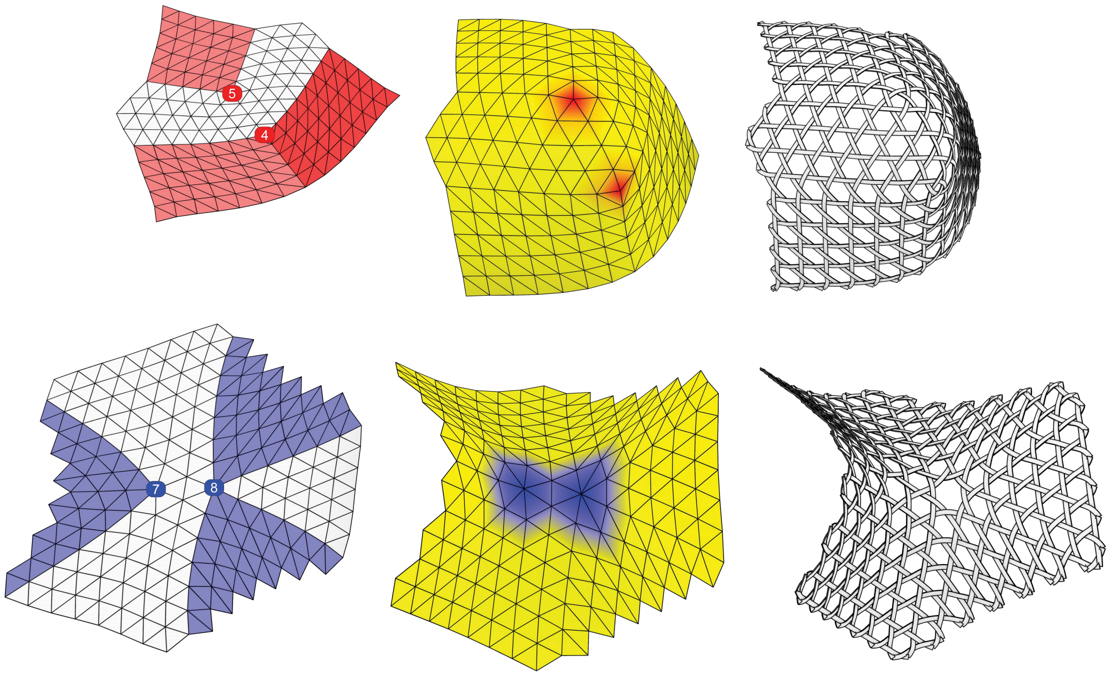

Figure 8: Introduction of singularities, relaxed mesh and interlaced weave representations for (**a**) $v4$ and $v5$ valence changes (top row) and (**b**) $v7$ and $v8$ valence changes (bottom row).

3.4 Case study 2 – multiple singularities with region crossings

Curating the introduction of singularities to ensure no crossing of the flip-regions is both tedious and limiting. In this case study, the non-crossing constraint is relaxed to examine the impact of edge flipping interactions across flipped regions. Here, the goal of introducing a singularity at a specified vertex whilst maintaining $v6$ regularity around it remains the same, but with the additional constraint that any previously introduced singularities are respected.

In allowing flip-regions to cross, it can become necessary to iteratively introduce singularities and perform a mesh relaxation after each introduction. This is due to the fact that mesh faces can become increasingly 'skinny', or even overlap through successive edge flips (**fig. 9**). With this amendment to the process, singularities can be introduced as before (**fig. 10**). However, certain configurations of singularities develop interferences, causing unwanted alterations to existing valence changes, as can be seen in **fig. 11**.

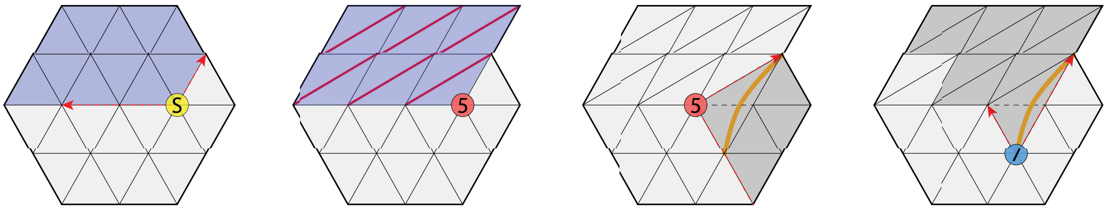

Figure 9: Successive edge flips can lead to overlapping mesh edges. Performing a planar mesh relaxation after each introduced singularity can mitigate this.

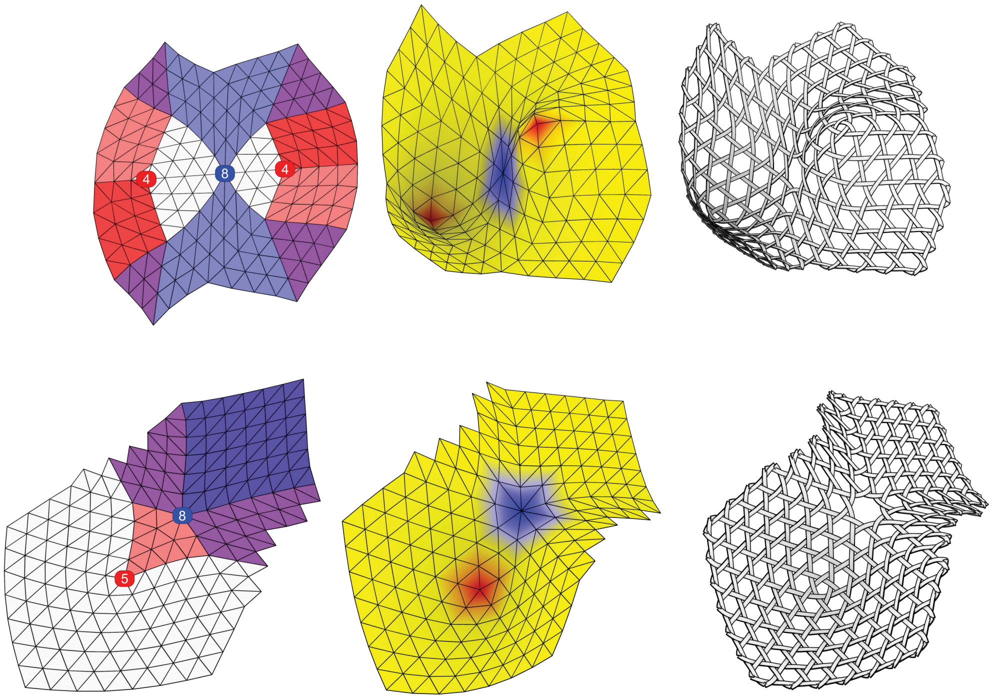

Figure 10: Introduction of singularities, relaxed mesh and interlaced weave representations for (**a**) $v4 - v8 - v4$ valence changes (top row) and (**b**) $v5$ and $v8$ valence changes (bottom row).

Two complimentary mitigation approaches can be employed to avoid negative interferences. The first approach is to iterate higher order valence changes within a single flip-region. For valence changes > 6, the valence is increased incrementally to the desired level and flip-regions are chosen to exist within the region of the previous increment. In this way, the mesh can expand with topological alterations contained within a more constrained portion of mesh (**fig. 12**).

This is in contrast to the multi-region adjustment method demonstrated with the singularity zoo, but employing the same topological adjustment principle. This approach is more challenging for valence changes that are < 6, as flipping iterations cause a contraction of the mesh. In the case of $v4$ and $v3$ valence changes, the iteration of flip-regions will consume 'fresh' portions of the mesh - which may contain previously embedded singularities (**fig. 13**). Nevertheless, this approach

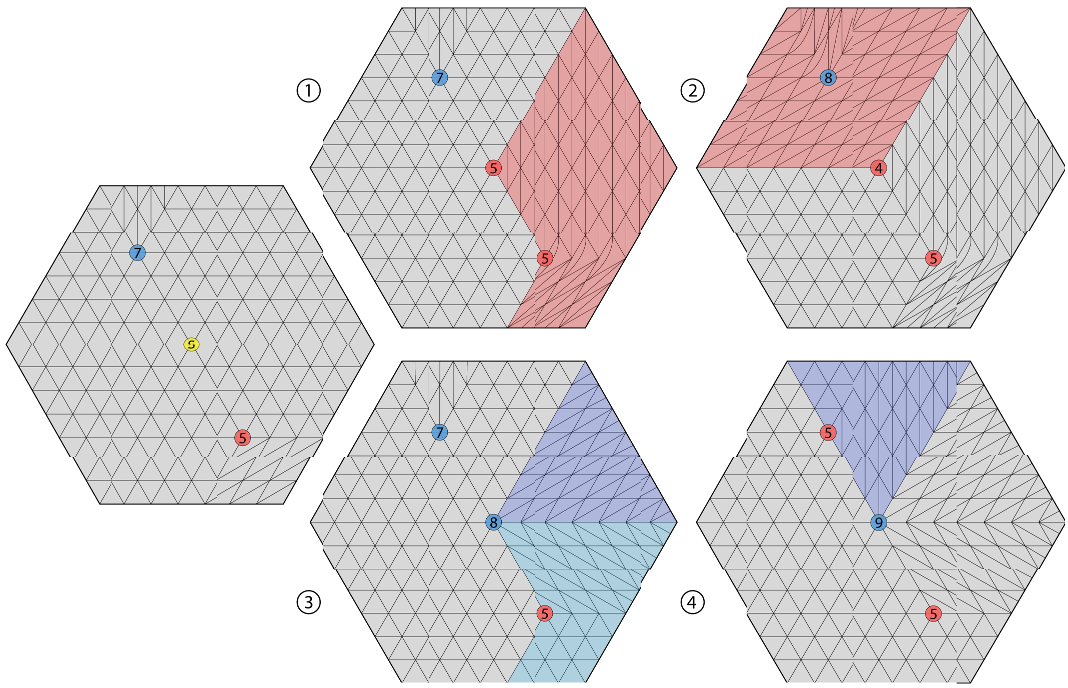

Figure 11: Study of the effects on existing singularities when introducing a valence change with region crossings. Cases 1 and 3 - introducing a $v5$ and $v8$ respectively has no effect on existing singularities. Cases 2 and 4 - introducing a $v4$ and $v9$ respectively causes an unintended change to the existing $v7$.

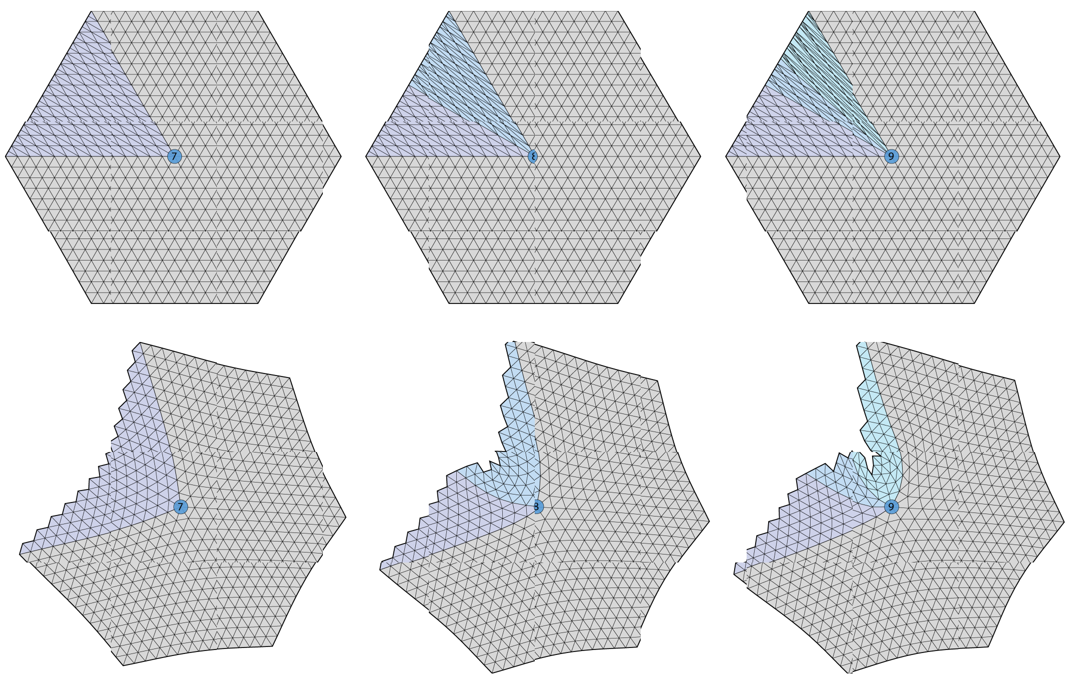

Figure 12: Higher order valences can be also be introduced using an iteration strategy. In this case, the valence is increased by one step at a time and within the territory of the first flip-region. In the case of > 6 valences this causes the mesh to expand when relaxing, as shown in the bottom row. This provides a useful strategy for designing singularity embedding sequences and helps mitigate issues of unwanted valences changes with certain region crossing interactions as shown in **fig. 11**

applied to lower valences still offers a reduction of effected territory in the mesh as compared to the multi-region approach shown in fig. 6. In both of these cases, the MTAS is extended to accommodate this iterative approach to introducing singularities.

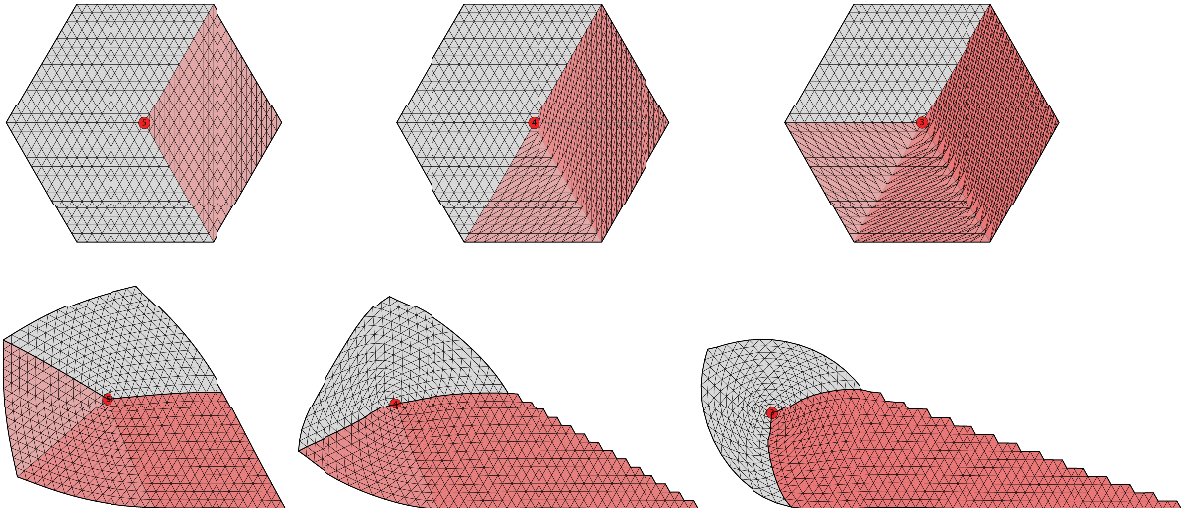

Figure 13: Applying the iteration strategy to valence cases < 5 results in a continued contraction of the mesh when relaxing, as shown in the bottom row.

The second mitigation approach is to carefully plan the introduction of singularities to avoid unwanted valence changes. This sequencing approach is not an automated aspect of the MTAS, rather it is dependent on the judgement and experience of the designer. Combining both interference mitigation approaches allows complex weave designs to be represented (fig. 14 and 15).

3.5 Case study 3 – basic digital design pipeline

This case study demonstrates the integration of the MTAS into a simple design pipeline that allows the user to interactively adjust a mesh and generate fabrication information. The fabrication information for each unrolled weaver includes indexed positions of the crossing points with other weavers, the interlacing direction of all crossing points (over or under) and a strip length breakdown that can discretise weavers to available material lengths (fig. 16). The discretisation includes an overlap distance to allow the components of the weaver elements to be spliced.

The interactive mesh adjustment requires user selection of target vertex and input of the desired valence. Singularities are introduced sequentially with mesh adjustment and relaxation updating in realtime. The user can switch between visualisations of the topology mesh, which provides additional false colour curvature feedback, or the interlaced weave. The interlaced weave is generated using the methods reported in the literature (Ayres et al. 2018, 2020). In this case, weavers have

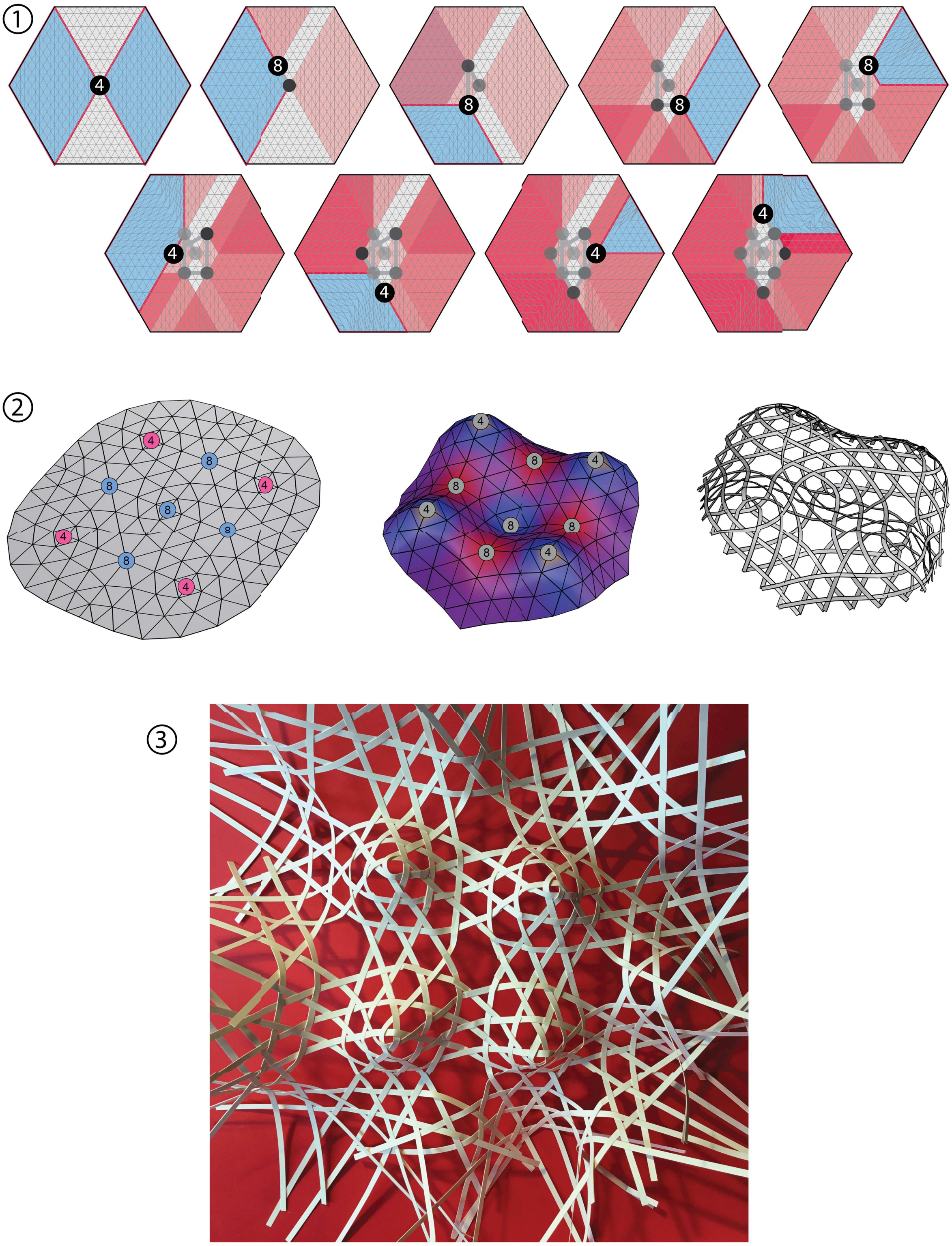

Figure 14: With coordinated sequencing of singularity introductions, unwanted valence changes caused by interactions of flip-regions can be mitigated as shown in top row (1). Center row (2) shows the topologically adjusted mesh, relaxation and weave resulting from the singularity sequencing shown above. Bottom frame (3) shows a physical weave for comparison.

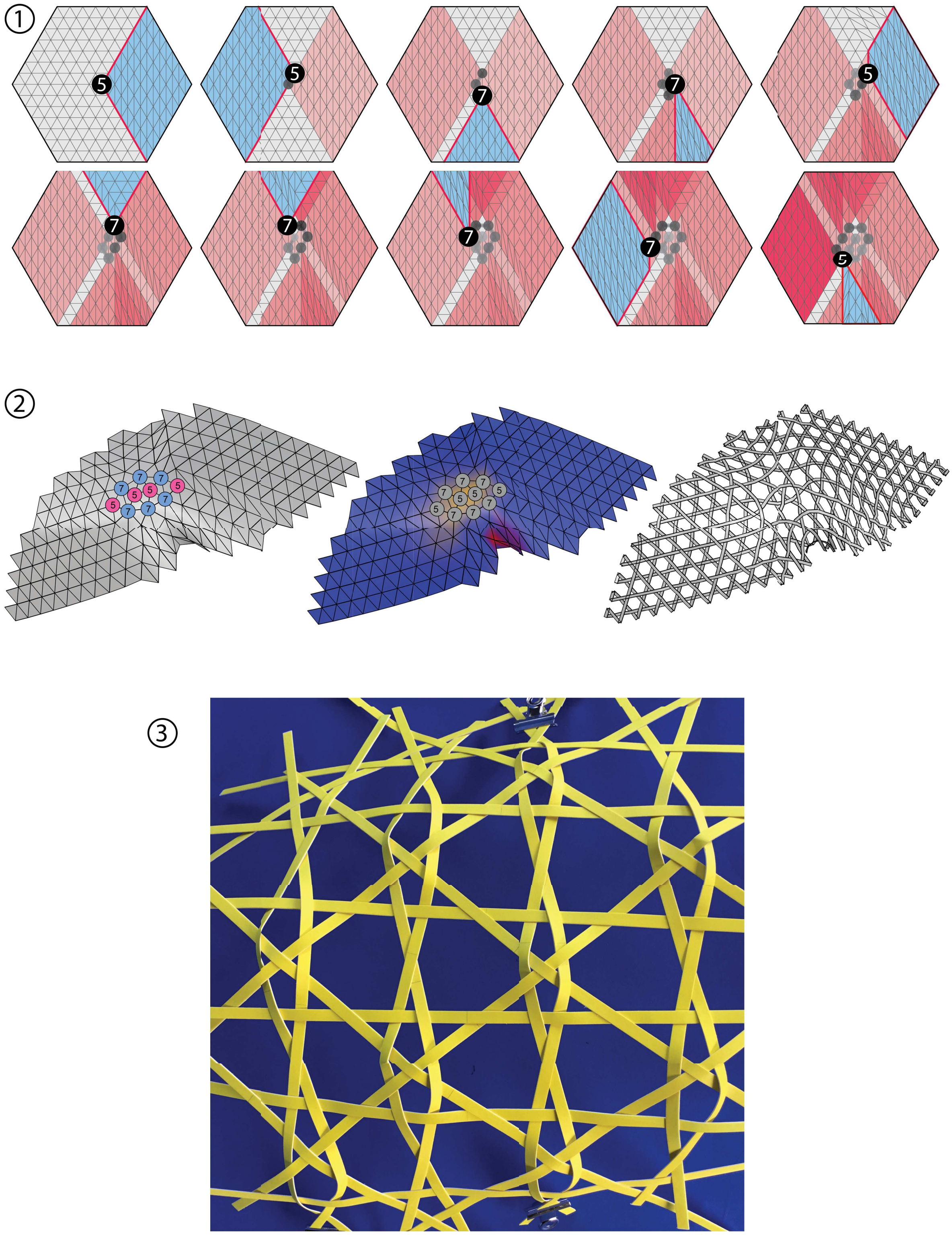

Figure 15: A second example with singularity introduction sequence (1). Center row (2) shows the topologically adjusted mesh, relaxation and interlaced weave. Bottom frame (3) shows a physical weave for comparison.

a circular cross-section by default. This can be altered to represent strips, with control over thickness and width. In both cases, adjustment of weaver cross-section dynamically alters the interlacing height. Achieving a balance between interlacing distance and height relative to cross-section size is, at present, reliant on visual judgement. However, computational approaches for dynamically finding weaver sizes based on line-line collision testing have been demonstrated in the modelling of biaxial weaves (Vestartas et al. 2018). A similar scheme could be implemented here in future developments.

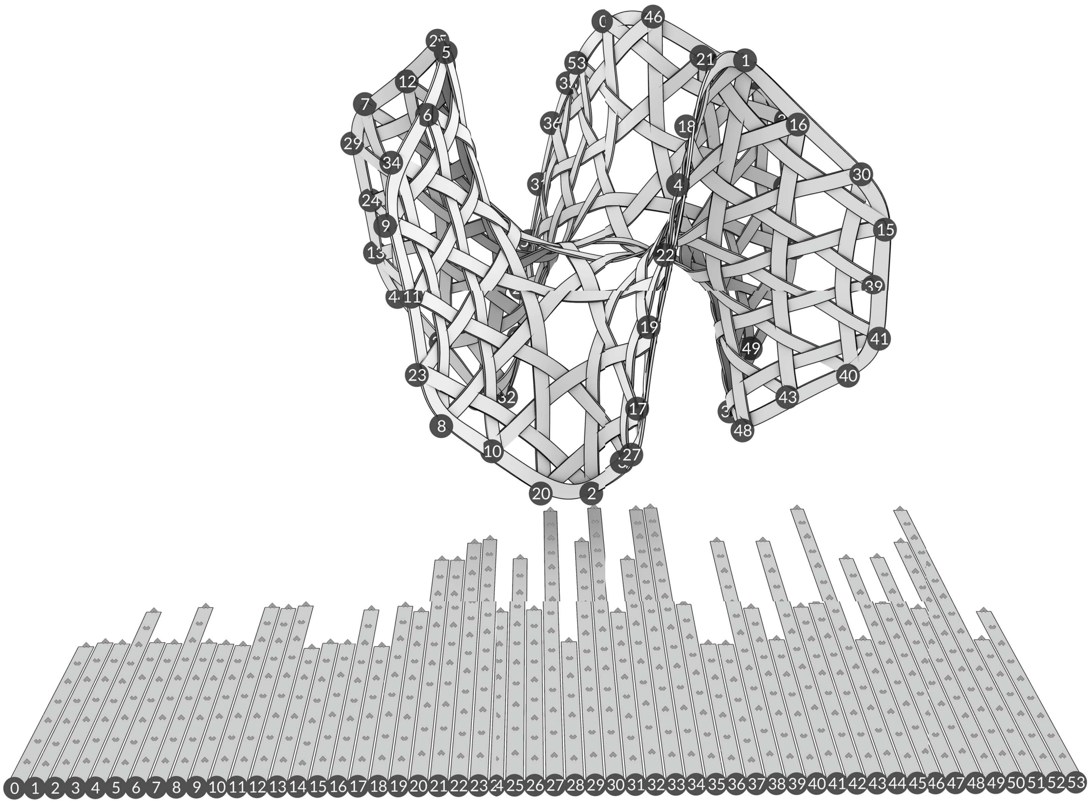

Figure 16: Result of the MTAS embedded in a digital design pipeline producing fabrication information for Kagome weaves.

4 Results and reflection

4.1 Results and limits of the MTAS

The mesh adjustment principle of using edge flipping within defined flip-regions has proven to be extendable in the task of embedding individual singularities from the target valence set. The valence set covers all possible lower valence conditions that are physically weaveable ($v3$, $v4$, $v5$), together with the practical higher valence conditions ($v7$, $v8$, $v9$).

In the case of pattern designs involving multiple singularities, the limits of embedding these without crossing flip-regions has been discussed. Nesting was proposed as a strategy for extending this approach but with clear constraints on legal valence introductions - limited to $v5$ and $v7$. Allowing flip-regions to cross and interact opens the design space but also reaches limits with certain configurations of singularity causing unintended valence changes at previously embedded sites. Mitigation strategies that complement the edge flipping region principle, provide methods for resolving many of these conflicts and allow for complex and dense singularity configurations to be realised.

The work presented here consciously limits the investigation to the adjustment of planar regular tri-meshes. The broader open challenge remains of developing a generalised Kagome pattern valence adjustment approach for planar and volumetric manifold meshes. A possible approach to this broader challenge might question the use of a regular $v6$ mesh as a starting point, instead, seeking to first establish a graph composed from singularity locations.

4.2 Limits of design integration

The work presented here has focused on describing and demonstrating principles and methods. The third case study demonstrates the integration of the scheme within a design pipeline of limited scope. In this case, modification of the mesh occurs through explicit inputs of singularity sites and valence, provided either as a list or through design interaction. While this does provide a useful and intuitive way of interacting with the system, there are clearly additional ways in which the MTAS could be integrated that would extend functionality and enrich modes of design engagement. For example, design surfaces could be provided as an input. This would require the development of an analytical engine to assess curvature with consideration of weave dimensions, then determine singularity sites and map valences to generate the weave. Extending further, weave topologies could be generated and manipulated from interaction with scalar fields, driving a performance oriented mode of ‘topology finding’.

5 Conclusion and outlook

In this paper, we have presented a mesh topology adjustment scheme specifically targeting the task of modifying a regular planar tri-mesh to produce Kagome weave patterns exhibiting double curvature. Curvature inducing singularities are introduced into the regular mesh by adjusting valence at the desired mesh vertex. It has been demonstrated that individual adjustments from the valence set $[v|3 \leq v \leq 9]$ can be successfully introduced using an edge flipping method. A key insight has been that edge flipping must be iterated using a 'burning front' approach to remove collateral valence adjustments. This creates flip-regions that extend out to the mesh boundary. It has been further demonstrated, through two case studies, that this scheme can be used to embed multiple singularities. However, it has been found that when certain flip-regions cross, flip interactions cause alterations to intended and previously embedded valences. Two mitigation strategies have been explored and demonstrated; these are: 1) in cases where valence changes are <5 and >7, to iterate the valence adjustment and locate these across or within the same region respectively; 2) to plan the sequence of singularity introductions. Combining both these strategies allows for complex fields of singularities to be introduced into regular planar meshes. However, limits to the MTAS are still to be found, for example with dense or extended fields of singularities which can, in certain circumstances, result in valence adjusting interference. As such, it cannot be claimed that the MTAS, in its current form, provides a 'cast-iron' generalised approach to regular planar mesh adjustment for the representation of any arbitrary Kagome pattern. Nevertheless, the state-of-the-art has clearly been advanced, useful patterns demonstrated, insights generated and limits determined - which provides resources for spurring continued research in this territory. Future efforts will also approach the more general and open challenge of topology adjustment for Kagome pattern representation with volumetric meshes.

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