Corrugated Cardboard Shell

A pavilion project of an architectural workshop

Lina Vestarte[1,*], Petras Vestartas[2], Romualdas Kucinskas[1]

[1] Department of Architecture, Vilnius Academy of Arts (VAA)
01124 Maironio st. 6, Vilnius, Lithuania

[2] Laboratory for Timber Construction (IBOIS) École Polytechnique Fédérale de Lausanne (EPFL)
IBOIS, ENAC, EPFL, 1015-Lausanne, Switzerland

[*] Corresponding author e-mail: linavestarte@gmail.com

Abstract

This paper presents mesh-discretisation and assembly methods for quad-based shell-structure. The study case is a corrugated cardboard shell 3m in height and spans up to 6m in diameter and is supported at 5 arbitrary points. The structure was built from-two layer 3mm corrugated cardboard sheets, consisting of 391 sets of planar quadrilateral elements (1537 total count of unique pieces). Since the principal fabrication constraint was a 2-axis laser cutter, the approximation of cutting angle for finger-joints was required. The laser cutter, with the maximum bounding area of 600 x 900 mm, also restricted the size of the panels, while the directionality of corrugated material posed constraints on geometrical proportions of the panels and their orientation within the cutting sheet. The shell was assembled without using heavy scaffolding or any adhesives, thus relying on material strength and friction provided by the connection method. The assembly sequence and insertion vector had a crucial role in the act of such construction. The pavilion was assembled within less than 12 hours, including the time spent for sorting the unique pieces.

Keywords: Corrugated Cardboard, Assembly sequence, Shell structures, Form-finding, Breadth-first-search, Polygonal meshes, Finger joints

Figure 1: The pavilion made from corrugated cardboard displayed in the art exhibition in 2019.

1 Introduction

1.1 Shell structures

Shell structures are constructed systems described by three-dimensional curved surfaces, in which one dimension is significantly smaller compared to the other two. They are form-passive (relatively rigid) and resist external loads predominantly through membrane stresses (Sigrid Adriaenssens and Williams 2014, Chapter 3). Typically made of timber (Robeller et al. 2016), concrete or masonry (Block et al. 2017). The unique membrane performance allows us to build efficient shells even from very cheap and/or weak (with regards to bending) materials, such as corrugated cardboard sheets, selected for this project.

1.2 Geometry

The challenge of shell design is to find the appropriate form for a given problem and since shell structures are considered as one of the most complex structural systems, it is very difficult to derive structural equations and sometimes impossible to solve them manually. Due to the fact that the structural behaviour of shells can be so complicated, the design methods of these complex structures strongly relies on model prototypes and testing (Sigrid Adriaenssens and Williams 2014, Chapter 4). Therefore, exclusive attention is given to the use of digital form-finding tools (Piker 2013; Rippmann et al. 2012) and physical modelling. Form-finding methods are the ones, where the structure itself defines its own shape based on its equilibrium under applied loads. Unlike geometric forms, which are defined mathematically, or free-form shapes, which are generated without taking into account structural performance, form-found shapes rely on the structure and loads themselves for definition, thus making such shapes inherently more structurally efficient.

2 Background

The 1:1 scale pavilion (**fig. 1**) is a result of a two-stage architectural workshop. The main idea for the construction method originated from a well known simple finger-joint technique, widely used from children's toys and furniture, to architecture. The teaching agenda for the workshop consisted of the following parts: introducing historical examples of form-finding, modelling with digital form-finding tools, getting acquainted with assembly sequencing and fabrication methods.

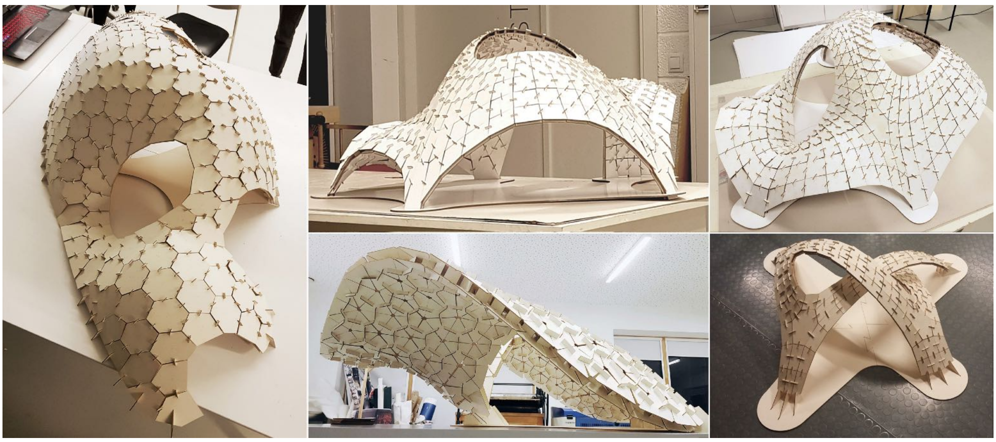

Figure 2: Students made physical models that were used for testing the form-finding methodology and the physicals parameters of the joint-scale, structural-thickness, panelling-topology and foundation.

In the course of the workshop, students were divided into groups to explore various mesh topologies (triangular, quadrilateral, hexagonal) within the realm of form-found double-curved shapes (**fig. 2**). Physical models were built as a two-layer system, from 2 mm cardboard using a laser-cutter. Each student group designed and fabricated supports, customised to suit their designs aesthetically and structurally. These physical prototypes served as crucial study examples, providing information regarding the dimensions and proportions for the design of the pavilion structure. The initial studies also exposed some important fabrication issues such as defining assembly sequence, solving insertion vectors per panel, and indexing, which was essential for building a larger scale structure.

During the second stage, the corrugated cardboard pavilion was built. This shell structure was 3m height and had a span of 6 m in diameter and was supported at 5 arbitrary points. The structure was built from two-layer 3 mm corrugated cardboard sheets, consisting of 391 sets of planar quadrilateral elements and a total of 1537 unique pieces. It was created using similar surface modelling methods as the smaller scale experiments conducted by the students, but using quadrilateral panels.

3 Methods: design phase

This project addressed following geometry processing methods as seen in fig. 3: mesh discretisation, digital form-finding, data-sets ordering for laser-cutting, and CNC fabrication and assembly. A dynamic-relaxation method was used for form-finding a discrete shell (Piker 2013; Deuss et al. 2015). Polygonal mesh processing tools were applied to: find a 2D topology-diagram, obtain planar elements, construct connecting elements with finger joints, and prepare the geometry for fabrication (Vestartas et al. 2020). A nesting tool OpenNest was used for orienting, packing, and labelling of the elements for fabrication.

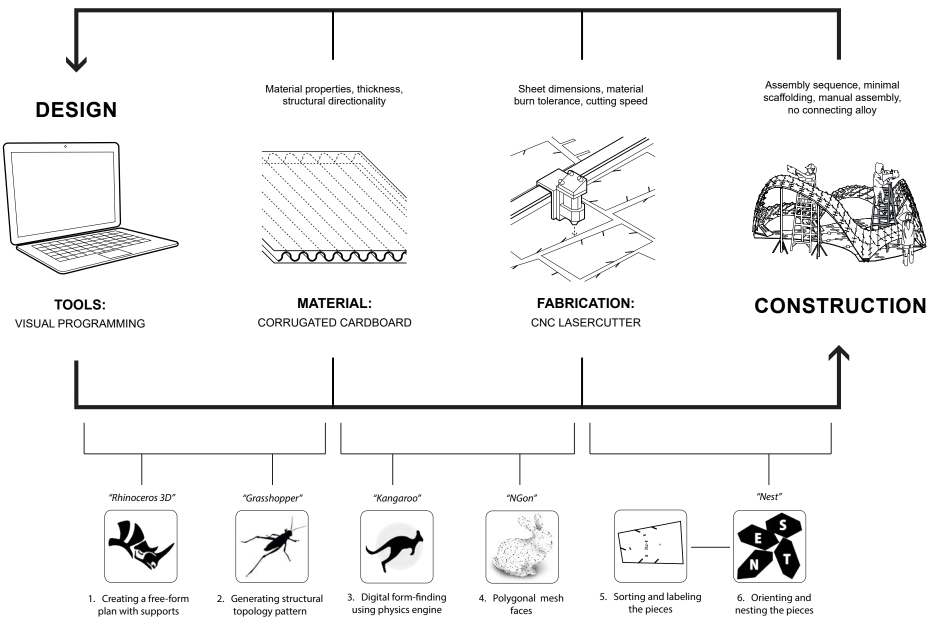

Figure 3: The workflow diagram presenting digital geometry processing tools, material constraints, 2D-Axis fabrication, and assembly.

3.1 Topology diagram

The first step in the design stage was to define two-dimensional outlines and generate a planar topology pattern within the given boundaries. Various methods for generating topology patterns were explored, when working with students on several smaller-scale models during the prototyping phase (**fig. 2**). Quad dominant block decomposition based on the medial-axis pruning/re-branching method (Oval et al. 2017; Oval 2019) was selected for proceeding forward to a larger-scale structure. Based on this methodology, the planar surface, represented by the boundary curves, was divided into a set of topologically simpler patches. Each patch was subdivided with quad pattern and then generated mesh was smoothed. The applied form-finding method computes discrete meshes or line networks with a list of geometry goals (Deuss et al. 2015). As a result, the selected topology pattern, an input to the form-finding process, is essential, since it is directly related to the obtainable geometries (**fig. 4**).

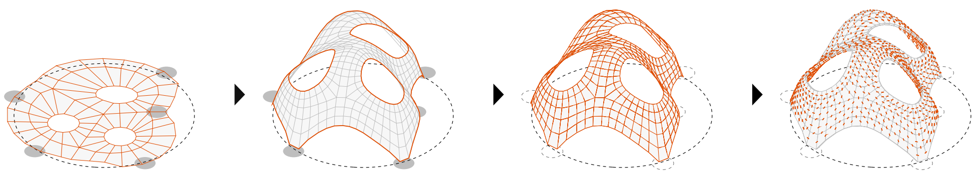

Figure 4: Design Workflow. (left to right) 2D topology diagram. Digital form-finding employing "hanging cloth reversed" model. A projection method for Planar Panels. Connecting elements for neighbouring panels.

3.2 Form-finding

The second part was to define boundary conditions and use form-finding. The selection and manipulation of anchor points play a major role in shaping the output geometry, thus being another significant constituent of the design scope. Digital form-finding was employed for the initial design phase using a stability or equilibrium (statics) approach, by simulating gravity load in the direction of Z-Axis. Students were acquainted with such methods first through the introduction to a "hanging cloth reversed" (i.e. fabric forming) model proposed by Heinz Isler, which proved to be a beneficial strategy in helping students to understand the underlying logic of such digital methods.

3.3 Projection method for planar panels

The planar mesh-faces are obtained by projecting their outlines to the polygonal mesh-face planes as illustrated in **fig. 5**. The projection method is based on a mesh where triangle mesh faces are grouped into quads, hexagons, or n-gons, represented by the RhinoCommon MeshNGon class. Triangle mesh-face properties such as faces and vertex normals and vertex-edge-face adjacencies (VEF) are used to tessellate a polygon internally. A triangle or quad, that is referenced in the polygon list, is no longer visualised and conceived as a single entity, which takes part in the NGon data-structure. Consequently, the normals and nine adjacency types (VEF) are reconstructed following the notion of a polygon. For instance, the plane of the NGon is computed by taking an average of the boundary-vertex coordinates and the mean of the triangle-face normals (**fig. 5c**). The polygonal mesh is initialised by triangulating an array of closed polylines using an ear-clipping algorithm (Eberly 2002), then joining and welding using the R-Tree search (Guttman 1984). The applied algorithms are detailed in the source code[1]. The projection function gets the closest-point to a plane (**fig. 5e**) and reconstructs the planar polygon.

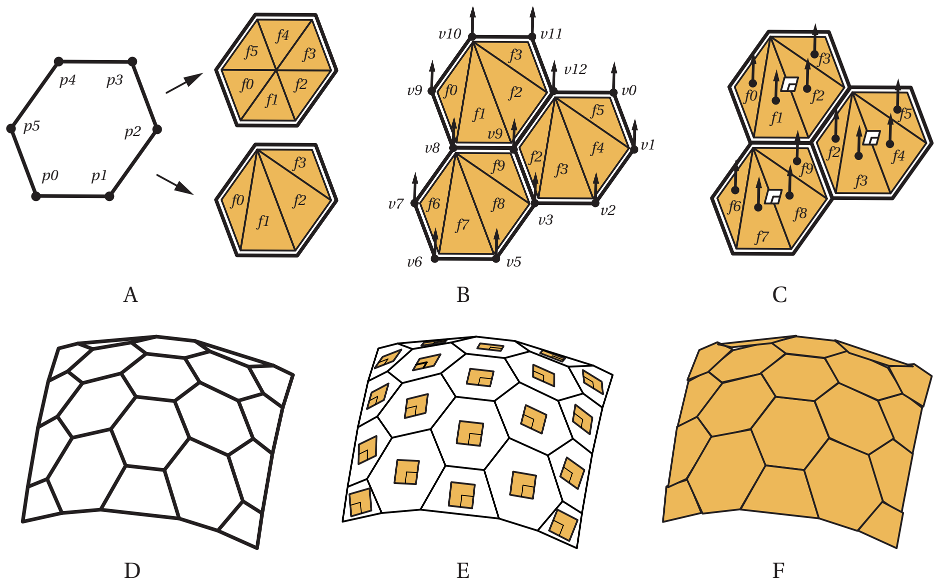

Figure 5: Polygonal mesh-data structure: (**a**) Triangulation of a polygon, (**b**) ngon mesh vertex normals , (**c**) mesh faces vertices and ngon planes. Projection method: (**d**) non-planar polygons , (**e**) average polygon planes, (**f**) planar polygons.

A second projection method could be used by intersecting polygon planes by bisector planes of the mesh edges, but this would only work for 3 valence meshes (Rippman et al. 2016; Pottmann et al. 2015). The geometry representation of the mesh-graph is no longer connected graphically, but the connectivity-graph is continued to be used from the non-projected mesh to construct joints, offset the mesh, and index the cardboard panels for the fabrication.

[1] Ear-clipping method: TriangulateOpenPolyline in MeshUtilSimple class and R-Tree Welding method: WeldUsingRTree in MeshUtil class (https://github.com/petrasvestartas/NGon)

3.4 A two-layer structure

The preliminary tests of the cardboard structure were made using one layer hexagonal panels and diagonal connecting elements as seen in **fig. 6**. After inspecting the single-layer test model, a two-layer structure was introduced to obtain greater stiffness and add resistance to twisting. The two-layer system was generated by offsetting polygonal meshes using their normals by the distance of their structural depth, taking into account the thickness of the sheet material (**fig. 7**).

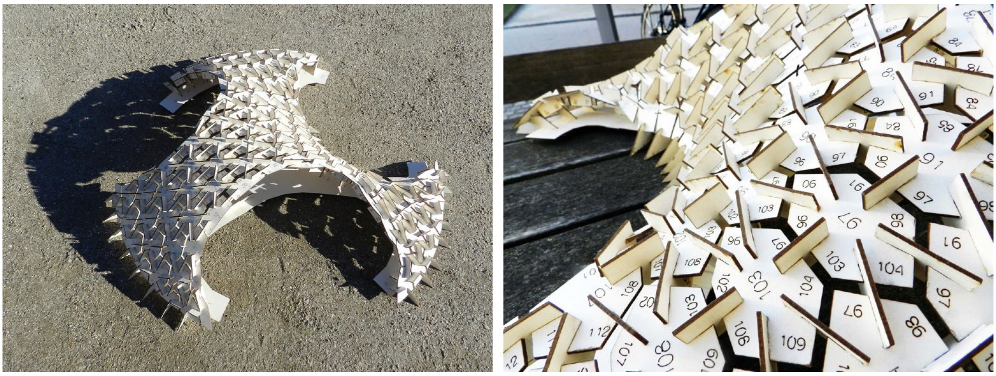

Figure 6: One layer cardboard model.

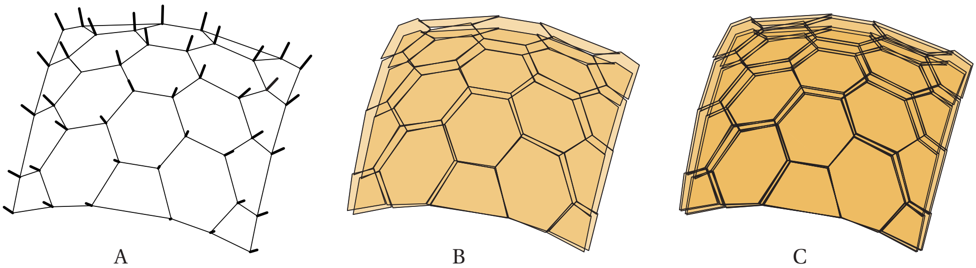

Figure 7: Mesh offset: (**a**) using vertex normals, (**b**) two layers of the projected mesh , (**c**) four projections considering the material thickness.

Several small scale models were made to check the right distance between the layers, taking material properties into account and the ease of the assembly. For the 1:1 pavilion structure, the distance of 9 cm between the two layers was selected, making the full structural depth (including the height of the connecting elements) of 15 cm. The experimental models were relatively small in scale and due to material elasticity, the assembly was possible without having defined assembly sequence and insertion direction. However, the issue of damaging the pieces during the insertion due to bending was exposed. Thus the larger structure needed the insertion vector and assembly sequence sorted. The equal distance offset was taken for further development as a straightforward solution, well suited for this particular project.

3.5 Diagonal connections

The diagonal connection elements with simple finger-joints were chosen to connect the pairs of panels in a double layer system, creating a closed cross-section. Several experiments were made to test the direction of the connection element. A connection perpendicular to the mesh edge performed well in transferring the loads but resulted in a greater risk of damaging the panels during the construction. The insertion direction was solved, finding one approximated vector per one set of panels (i.e. inserting both layers of the same enumeration at the same time). The sequence of the assembly, and thus the resulting insertion vector, could be specified either manually drawing a path (**fig. 8b**) on mesh-face centres, or using graph methods such as Breadth-First-Search as illustrated in **fig. 9** and the source code[2].

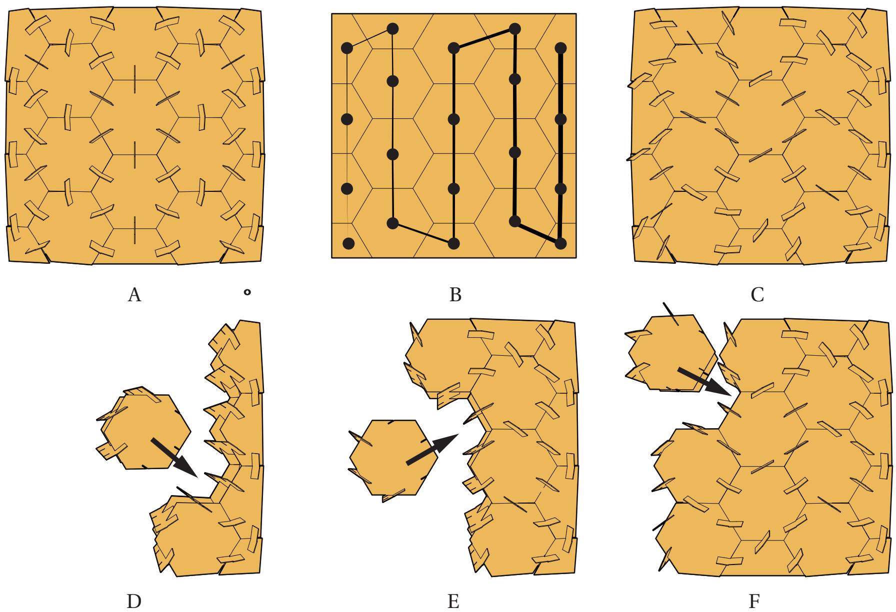

Figure 8: Insertion order following (**a**) perpendicular to edge (**b**) user specified insertion sequence (**c**) diagonal connections following the insertion order. (**d-e-f**) assembly sequence order showing the orientation of connections following one direction per set of panels.

The modelling of the connection pieces was based on a mesh edge and face adjacency. Each joint was placed on a planar mesh-edge, connecting the two neighbouring panels. A mean value of the two lines was taken since the panels had shifted slightly after the application of the projection method. The orientation of the connection element depends on the mesh edge direction and the sum of normal vectors of the adjacent mesh faces. In order to identify the orientation of the connection element during the construction, a design decision to shorten the connecting part outside

[2]Breadth-First-Search method for a Mesh: MeshBFS in UndirectedGraphBfsRhino class (https://github.com/petrasvestartas/NGon)

the double-layer structure was made. A series of tests were performed with smaller prototypes as well as a cluster test conducted in the original scale and material. This allowed defining the suitable proportions and dimensions for these elements. For the pavilion structure, the width of the interior parts of the connection pieces are up to 20cm, the outer part is 12cm and the total height is 15 cm. The larger outline arches on the edges were stiffened with a double diagonal connection and lengthened interior parts (**fig. 10a**).

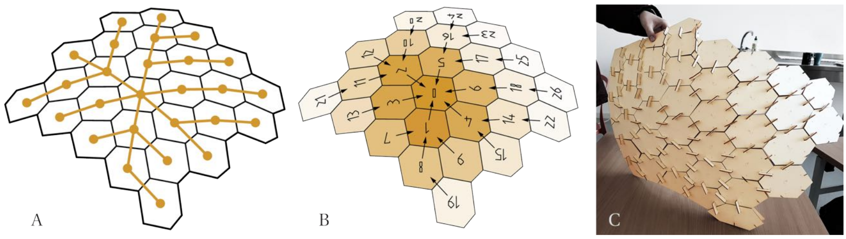

Figure 9: Insertion order following (**a**) Breadth-First-Search graph method (**b**) finding insertion vector (**c**) physical test model.

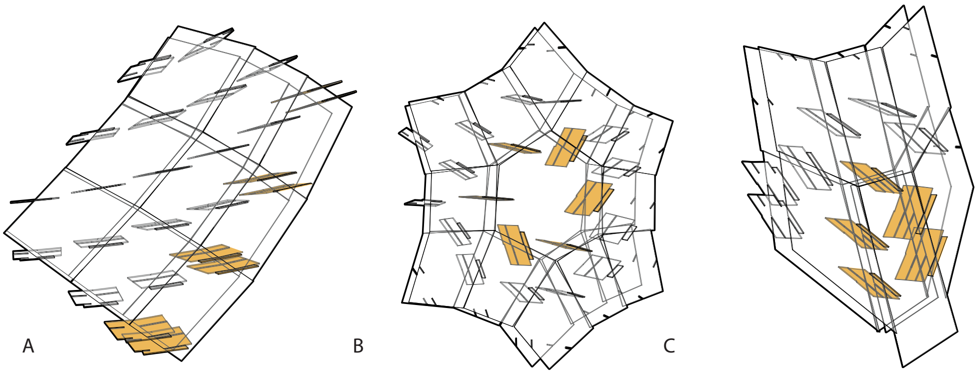

Figure 10: Details of finger joints (**a**) doubling of connections for the out arch (**b**) elongated joinery (**c**) foundation part with two and three layers of panels.

3.6 Finger joints

The traditional finger joint method used for such structure results in an angular cut through the material thickness. This is the case because of the inherent properties of double-curved surface geometry (**fig. 11**). Due to the main constraints being a simple laser-cutter and a thin sheet corrugated cardboard (3mm) material, an approximation for a two-dimensional cut was needed. Although performing best in compression, with well-managed friction within the joint, the traditional finger-joint can also resist comparatively small amounts of tension. In order to exploit the potential of friction within the joint, the offset tolerance of both the depth and width of the finger joint had to be defined after thorough physical trials. The most critical aspect during such testing was a precise definition of an actual burnt parameter for a specific material, using specific laser-cutting settings. This was done to find a balance between ensuring fast and easy insertion, while also providing enough friction for stabilizing the structure during the construction process.

Figure 11: View of the finger joints in the assembled pavilion model, showing one connection element per edge for inner panels and two elongated connections for the boundary edges.

4 Methods: fabrication

4.1 Clustering

During the initial prototyping phase, the panels with the associated diagonal connection elements were grouped into a series of clusters, to ease the fabrication, sorting, and assembly processes. This was done due to the management issues of large numbers of unique pieces. It also helped to divide the work among the groups of students, when clusters were assembled in a parallel mode, and afterwards connected. A K-Means clustering algorithm (Accord.NET) was used to partition elements by cartesian coordinates. This technique reflects on the cluster-seam connections because large groups were joined together using multiple joints at once. Such rapid connection was possible because of the relatively small scale of the physical prototypes. Within the context of a larger scale structure, grouping strategies were used for creating sets of panels grouped with the associated connecting elements. These small sets, consisting of only several items, were later used for speeding the fabrication, manual sorting process, and the assembly.

4.2 Orienting and nesting

The panels and connecting elements contained geometrical information of cutting outlines, planes, and engraving curves for indexing. The plane of the element was used to orient the properties of the panels from 3D space to 2D XY plane. The central cutting outline is approximated from 5-axis angles in the 3d model to 2d-axis laser-cutting. A nesting algorithm was applied for the small scale prototypes. The packing method kept track of the transformation matrices from XY-plane to the nested part to orient additional data such as engraving polylines. The engraving was performed to track the data of the neighbour faces on each edge, first and second layer index, the panel id and assembly markers. The small scale models were cut from several sheets of cardboard with many pieces nested together. The digital nesting method performed fast (less than a second) for a large number of elements running only one iteration of packing. With regards to the 1:1 pavilion structure, there was a significantly smaller number of elements per sheet to be nested, the genetic algorithm performed less precisely, otherwise the time to compute increased exponentially.

The material used to build the 1:1 scale pavilion, was a single wall corrugated board, consisting of a fluted corrugated sheet and two flat liner-boards. The geometrical proportions of panels had to be restrained due to the material properties, for instance, elongated pieces and sharp corners had to be avoided. The directionality of corrugation also strongly affects panel performance under stress in different directions. After physical testing, the decision was made to mix the directionality among the panels (**fig. 12**), ensuring that the same set of facets would have a varying corrugation direction among the inner and outer layer of the structure. This was achieved through controlled nesting of specific panels within the sheet with the intentionally set direction of corrugation. Single corrugation direction, the best performing in compression, was chosen for fabricating diagonal connection elements. The fabrication constraint regarding the laser-cutter bed with the dimensions of 600 mm x 900 mm, significantly reduced the number of panels per sheet for the pavilion structure, compared to the smaller prototype models. This resulted in extended fabrication time but was advantageous in speeding the process of sorting and packing the pieces.

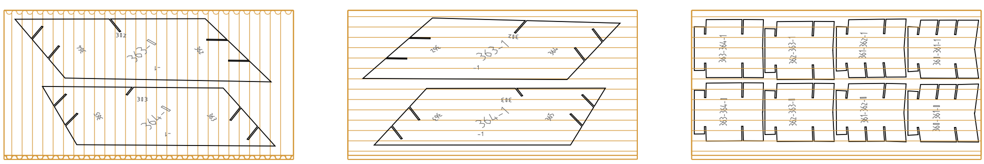

Figure 12: The orientation of elements considering the direction of a cardboard corrugation. The first layer panels of a two-layer system were rotated ninety degrees in relation to the second layer.

4.3 Sorting and labelling

Due to a large number of unique, but very similar pieces, the labelling was necessary for the assembly of both the test models and the pavilion structure. The latter consisted of 1537 pieces in total. During the construction of the pavilion, labelling also served for sequencing the assembly. The enumeration of the panel, according to the assembly sequence, was engraved in the centre of the panel and the orientation of the text followed the insertion direction (**fig. 13**). Each panel also had identification for the inner or outer layer and numbers naming the neighbouring panels, with regards to each edge. The diagonal connection element had a label marking the neighbouring panels that it connects. The fabrication of pieces followed the order of the enumeration. Both panels for inner and outer layers as well as connecting elements of a consecutive order were grouped for fabrication. These clusters, consisting of two panels and their associated connecting elements, were packed together for safe preservation and fast access during the assembly (**fig. 14**). Thorough labelling provided all the necessary information for the assembly, within the pieces themselves, ensuring fast and precise on-site construction. For aesthetic purposes, the labelling was hidden inside the structure, between the layers.

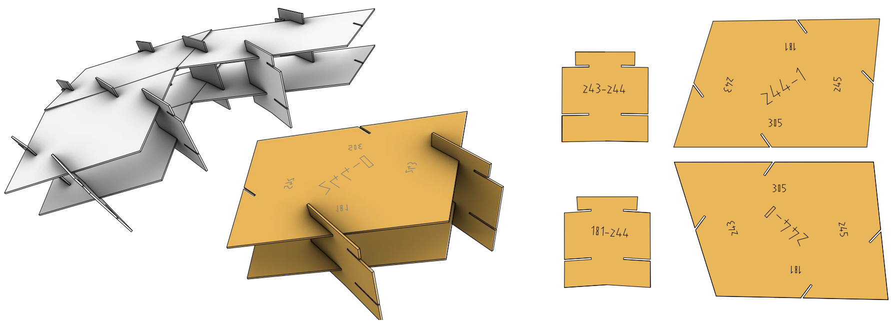

Figure 13: (**a**) A set of elements (two panels and connecting elements) and (**b**) the preparation for the assembly.

Figure 14: (**a**) Grouping the elements two panels and their connecting elements (**b**) packed pieces prepared for the on-site assembly .

5 Methods: construction

5.1 Assembly sequence

The assembly sequence directly corresponded to the enumeration order of the panels and consequently, the insertion direction. A linear path was chosen to connect the components starting from the ground and spiralling towards the top of the shell. The possibility to split the sequence and work parallel in several clusters was made possible when still maintaining the sequential order: starting from a larger enumeration number and adding the next set, marked with the successive smaller sequence number.

5.2 Assembly process

The base and the five supports were custom designed and fabricated using a 3-axis CNC machine from 18mm MDF panels. First, the base and the supports were put in place. Second, following the sequential labelling order, the five stiffened outline arches on the edges, that convey the loads to the supports, were assembled. Then, maintaining the assembly sequence, the structure was assembled in several clusters in parallel, merging in one assembly path towards the final pieces. The structural balance was secured during the construction using temporary supporting posts, but no need for a heavy scaffolding arose. This was due to the resistance to tension provided by the friction within the joint, allowing the possibility of small temporary cantilevers and compensating any possible instabilities occurring during the construction. The assembly process was fast and precise, the whole structure was built within less than 12 hours by a dozen students (**fig. 15**).

6 Results and reflections

The resulting pavilion project (**fig. 16** and **17**) provided some fruitful insights considering possible future investigations, regarding the assembly aspects in particular. The direction of connection elements that followed one insertion vector per one item (two panels and their connections) fostered some major challenges. The presented study was based on extensive physical prototyping and testing fabrication tolerances, thus it could be highly beneficial to use computational optimisation strategies to find the best-balanced solutions taking into account the assembly sequence, insertion vector and the distribution of loads within the shell structure. An additional aspect worth investigating further could be the design of the assembly sequence, employing certain clustering algorithms while taking into consideration off-site construction and speeding up the construction process in general. This approach would require a different type of joints for connecting multiple elements across each cluster.

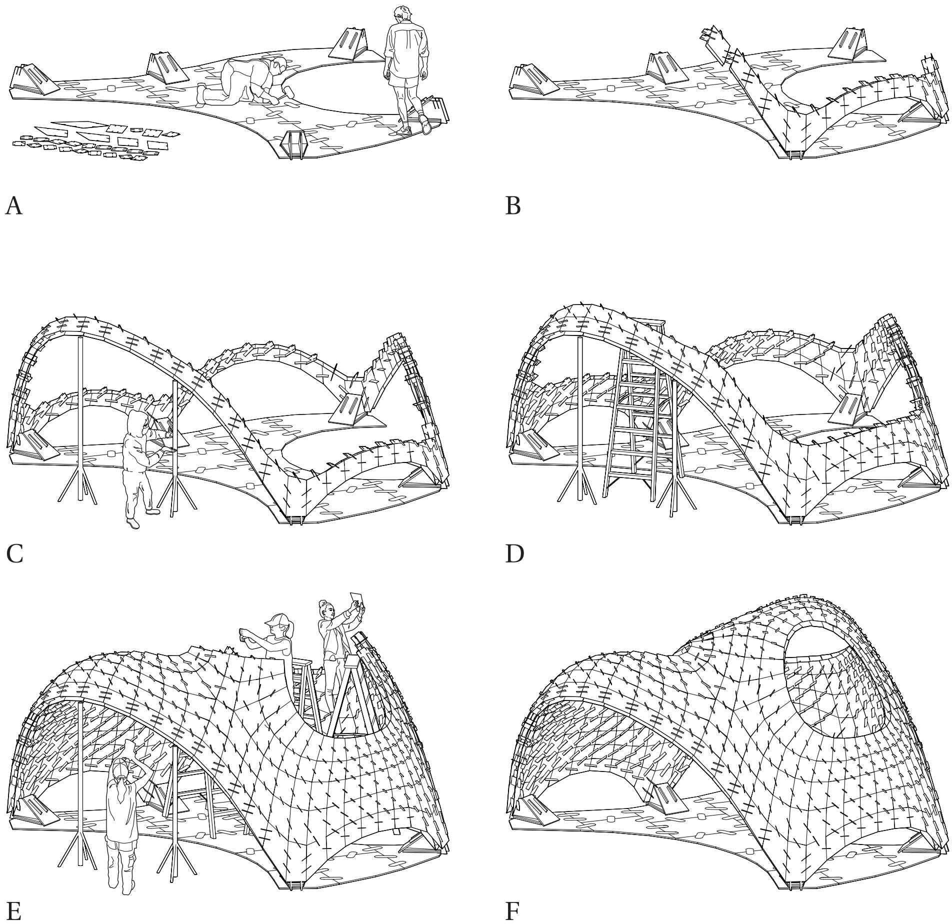

Figure 15: The assembly process (**a**) started from the base with five foundation elements (**b**) then the largest boundary arch was assembled (**c**) which required temporary punctual supports. The assembly (**d**) followed a spiralling insertion order (**e**) with multiple students assembling the proposed cardboard pavilion (**f**) .

Figure 16: Different views of the pavilion project in the exposition space.

During this study, students were introduced to digital design strategies and various visual programming tools with more explicit emphasis on physics simulation tool for form-finding and mesh-processing. Next to digital modelling, physical prototyping leads to recognizing and analysing subtle nuances in precision and tolerance, which helps to understand crucial distinctions between the digital models and the physical ones. Although the experimentation phase was a great learning opportunity, much appreciated by the students, having some "hands-on" building experience with a larger structure proved to be a better lesson. Constraints of fabrication, logistics, and material properties amplified by scale, indisputably proved the importance of incorporated structural thinking and the act of construction, in the preliminary design phase for such architectural structures as shells.

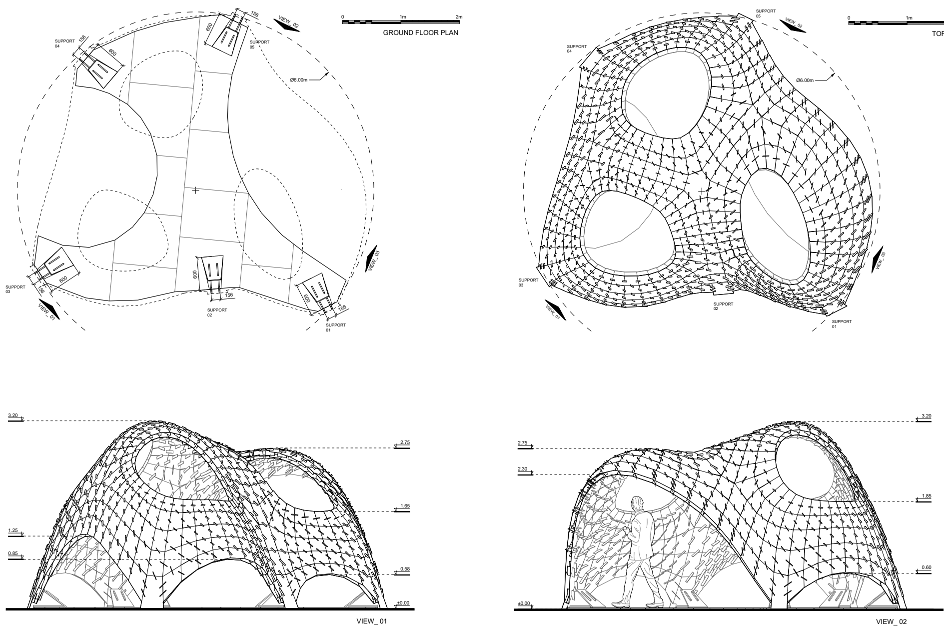

Figure 17: Ground floor, top view and elevation drawings.

7 Conclusion

This paper discussed design and fabrication strategies for polygonal shell-structures using digital form-finding. It presented a study case of a quad-dominant corrugated cardboard shell as a result of an extended workshop conducted with architecture students. The pavilion was demonstrated in the indoor art exhibition to showcase the studio-teaching. The structure was designed to suit the given material - corrugated cardboard and the fabrication method - 2D-axis laser cutter, making these two constraints as the main design scope constituents.

Acknowledgements

We are grateful to The Vilnius Academy of Arts (VAA), Department of Architecture, where the workshop took place in 2019, and to all the students who participated: Agne Avulyte, Evelina Azuolaityte, Zivile Bakutyte, Vilte Bernataviciute, Raminta Bieliauske, Agne Brazinskaite, Solveiga Buozelyte, Zivile Grigaleviciute, Rugile Jonaityte, Migle Karaskaite, Liucija Kuchalskyte, Jovita Laurukenaite, Kseniya Liadnik, Karolina Masalskaite, Emilija Minikaviciute, Alisa Muchks, Simonas Pinkoraitis, Kiril Smotkin, Ieva Sinickaite, Vytenis Stasiunas, Emilija Staskute, Justina Tautvydaite, Algimantas Urbonas, Rokas Vainiunas, Milda Venslaviciute, Ula Zebrauskaite, Kasparas Ziliukas.

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